On the Exact Analysis and Design of Dual-Hop, Maximum End-to-End SNR Relay Selection, and Full Selection Dual-Hop AF Systems

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North Pole
Polar Ice
Russia
Arctic Ocean
North Magnetic Pole
Ellesmere Island
Alert
Kalaallit Nunaat
(Grønland)
(Denmark)
Norwegian Sea
Norway
Greenland Sea
UK
Iceland
Bering Strait
Arctic Circle
Alaska
(USA)
Beaufort Sea
Banks I
Resolute
Baffin Bay
Davis Strait
Inuvik
Dawson
Great Bear Lake
Victoria Island
Gjoa Haven
Baffin Island
Iqaluit
Yukon
Whitehorse
Mackenzie River
Northwest Territories
Nunavut
Baker Lake
Yellowknife
Great Slave Lake
Labrador Sea
Atlantic Ocean
Ungava Bay
Newfoundland and Labrador
Nain
Fort Nelson
British Columbia
Prince George
CANADA
Kuujjuaq
Fort McMurray
Alberta
Saskatchewan
Churchill
Hudson Bay
Churchill Falls
St John's
Pacific Ocean
Fraser River
Edmonton
Manitoba
L Winnipeg
James Bay
La Grande Rivière
Sept-Îles
Saint-Pierre et Miquelon
(France)
Vancouver
Kamloops
Calgary
Saskatoon
Sandy Lake
Quebec
Prince Edward Island
Charlottetown
Victoria
New Brunswick
Nova Scotia
Regina
Winnipeg
Thunder Bay
Ontario
Québec
Fredericton
Halifax
Montréal
United States of America
Ottawa
Lake Superior
Toronto
Lake Michigan
L Huron
Lake Ontario
Lake Erie
LEGEND
National capital
Provincial or Territorial capital
Other populated places
International boundary
Provincial or Territorial boundary
Exclusive 200 nautical mile Economic Zone (EEZ)
Scale
km 300 0 300 600 900 km
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Outline

1. Introduction
2. System and Channel Models
3. Dual-Hop AF Systems
4. Maximum End-to-End SNR Relay Selection
5. Full Selection Dual-Hop AF Systems
6. Conclusion

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Impairments in Wireless Networks

1. Path loss

Dissipation of the power radiated by the transmitter as a function of the distance (d) traveled by the radio wave

$$\frac{P_R}{P_T} \propto (d)^{-\alpha}$$

P_R - received power
P_T - transmitted power

α is the path loss exponent

Environment	**Typical α Range**
Free space	2
Urban microcells	2.7-3.5
Office (Same floor)	1.6-3.5
Office (Multiple floors)	2-6
Home	3

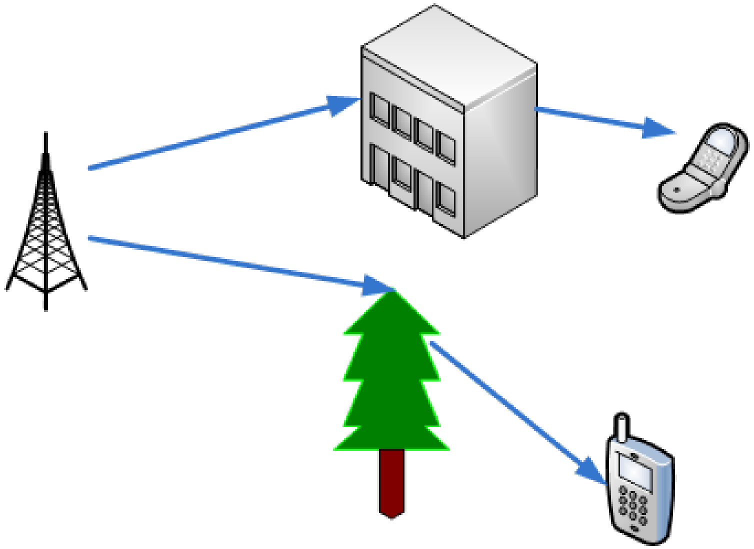
Impairments in Wireless Networks

2. Shadowing

Attenuation of the signal power by obstacles located between the transmitter and the receiver

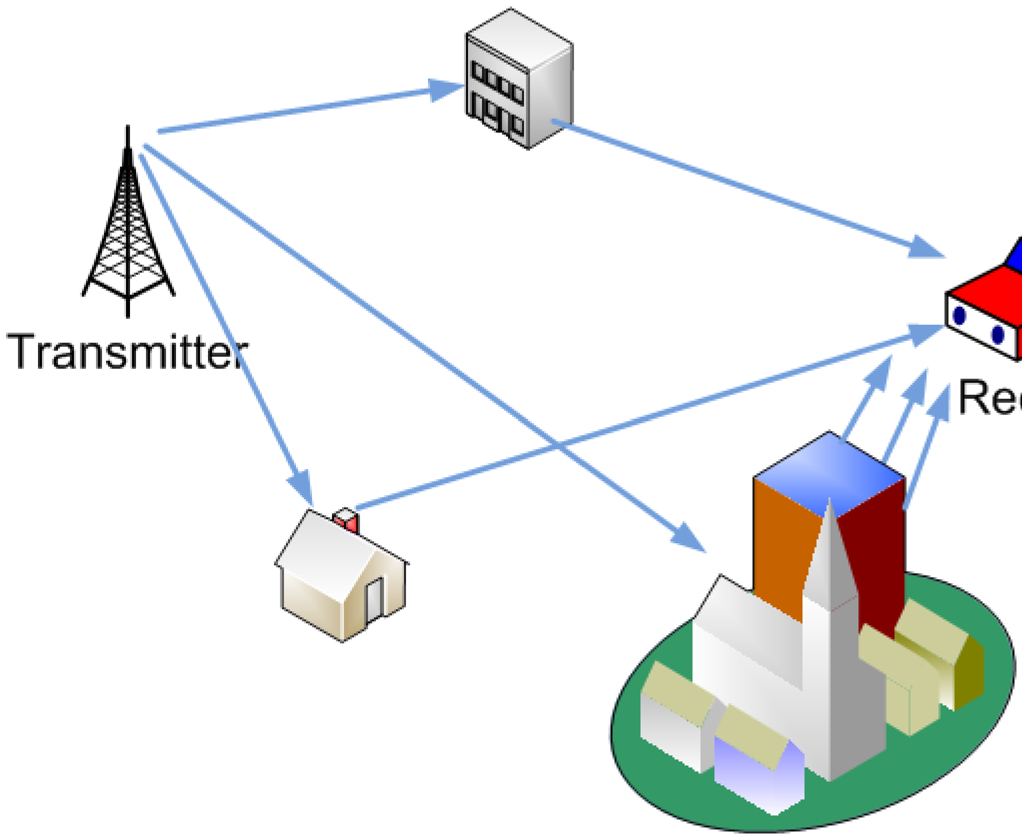
» E.g.: Trees, buildings, hills, etc.

Due to shadowing, actual received power is different from the power level predicted by path loss model



Impairments in Wireless Networks

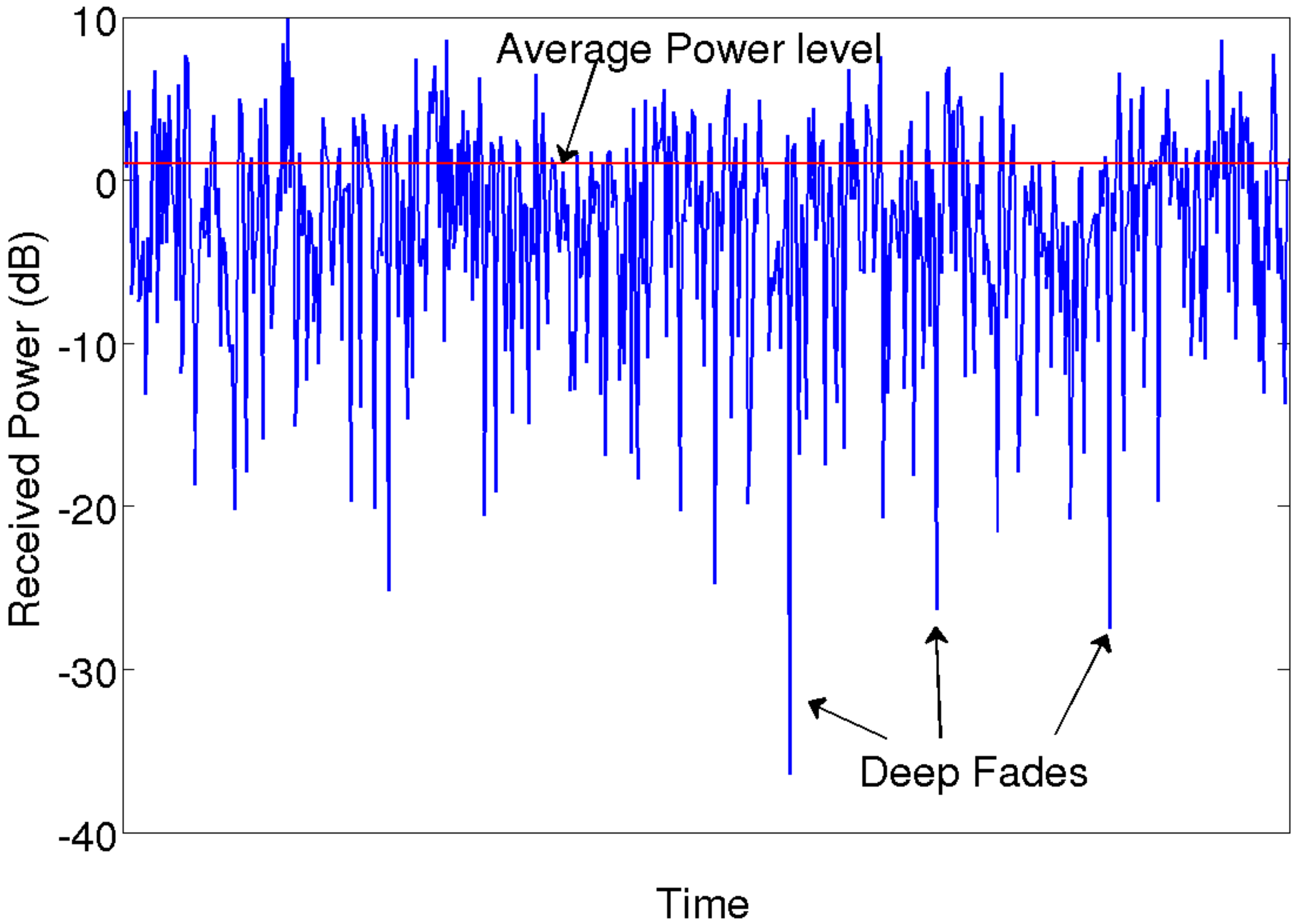
3. Multipath fading



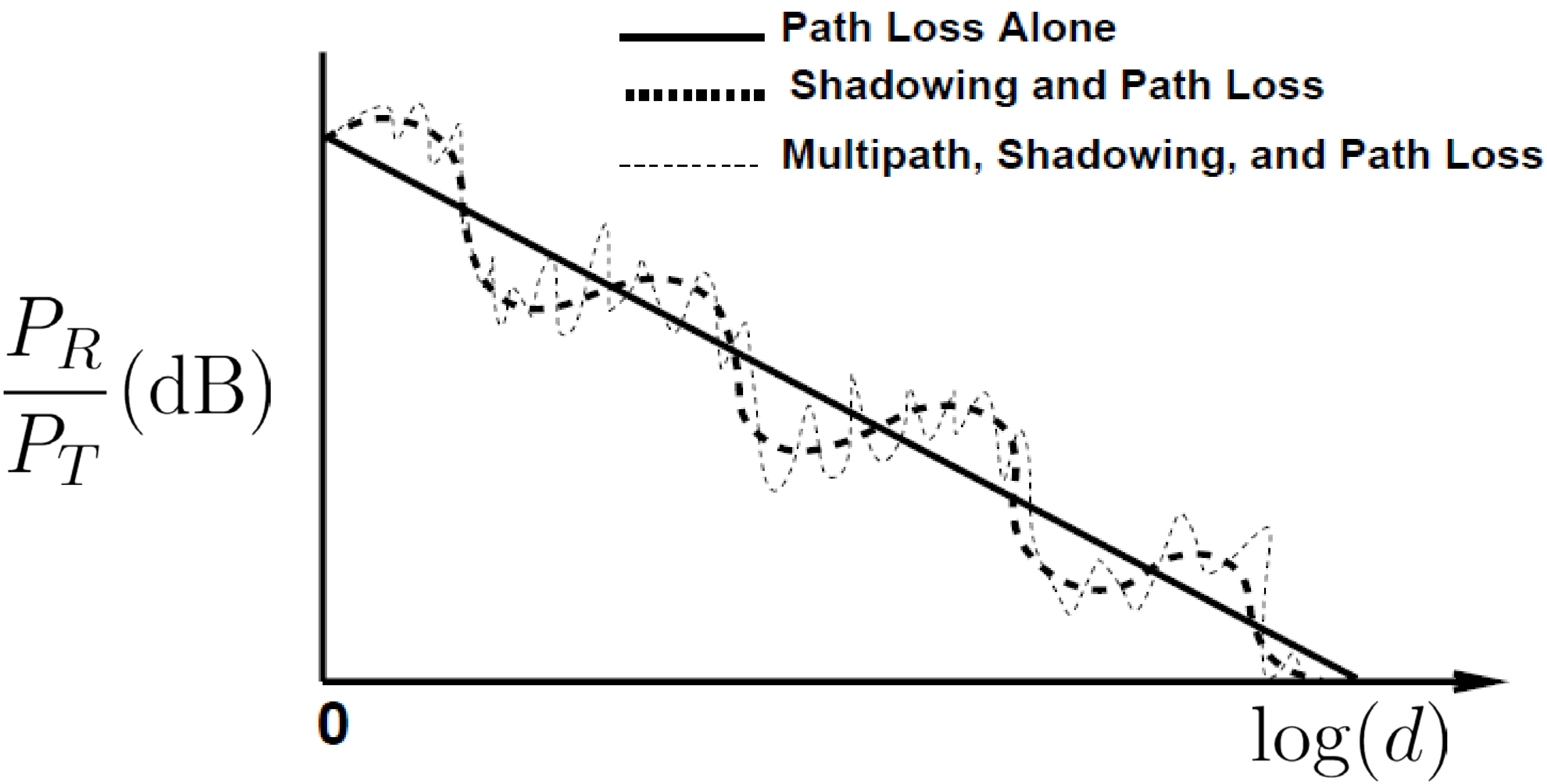
Multipath Fading

- Receiver may receive the same signals via multiple paths due to reflections by various objects along the way
- These multipath signals may add up constructively or destructively (deep fade) at the receiver
- Combined effect of **multipath signals** and the **motion of the transmitter and/or the receiver** results in **random fluctuations of signal power** at the receiver
- Signal amplitude and phase at the receiver is modeled using probability distributions
    - E.g., Rayleigh distribution for amplitude and uniform distribution for phase

Multipath Fading



Impairments in Wireless Networks



[Goldsmith 2005]

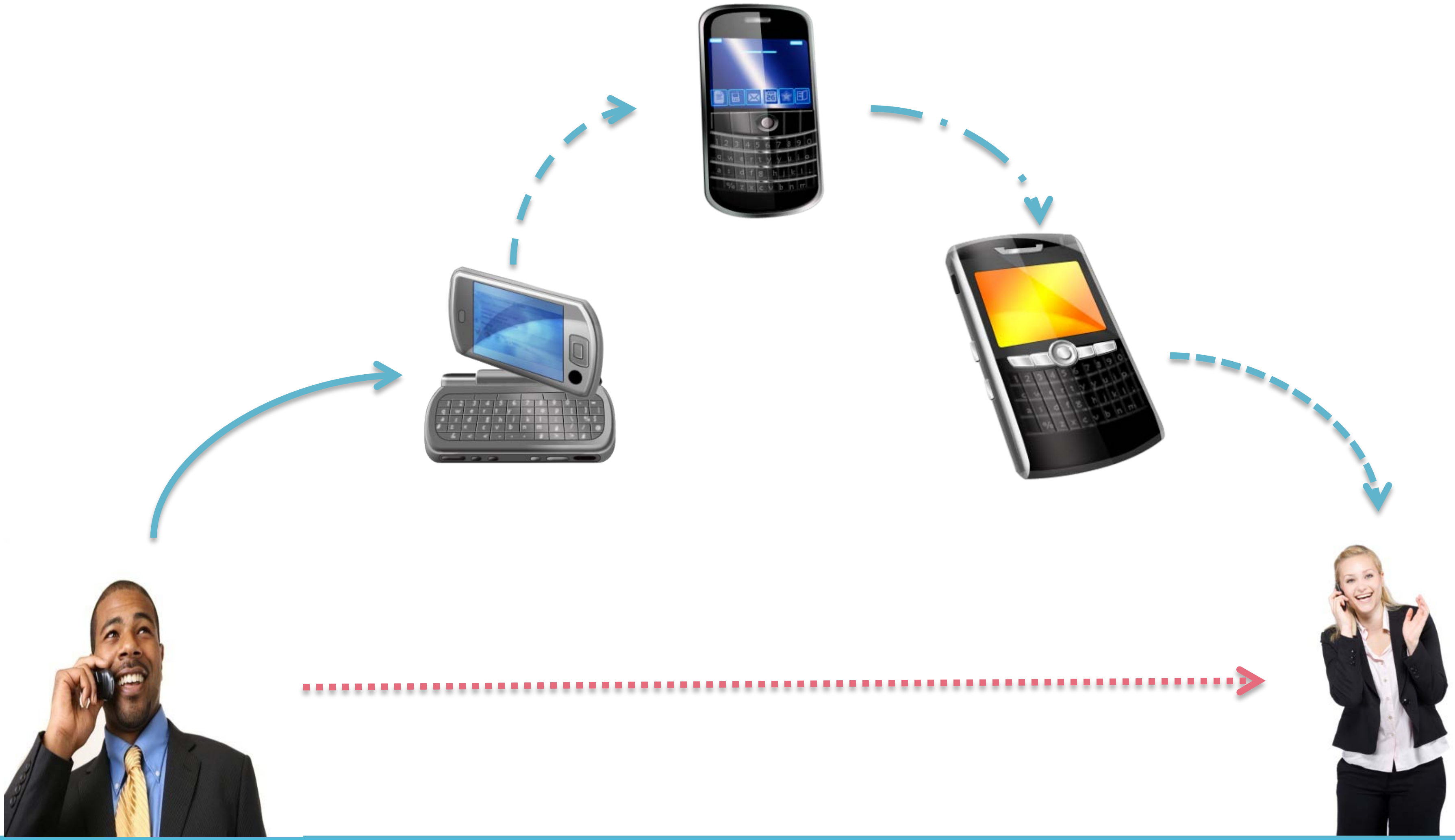
Solutions: MIMO

- Using **multiple antennas** at the transmitter and/or at the receiver
  - Enables signal transmission over **multiple independent channels**
    - Reduces the probability of occurrence of a deep fade
  - Leads to **diversity gain**

Diversity Gain - Number of independent channels that must be in deep fade to have the entire transmitter-receiver channel be in deep fade

But having multiple antennas at the source and/or destination **does not mitigate shadowing**

Solutions: Cooperative Networks



Dual-Hop Cooperative Networks

Cooperative Relay Networks

- Geographically distributed relay nodes are used to create a virtual antenna array

- Advantages of cooperative relaying include
  - Diversity gain (independent fading paths)
  - Coverage extension (relays amplify the signal power)
  - Mitigation of shadowing (relays are geographically distributed)

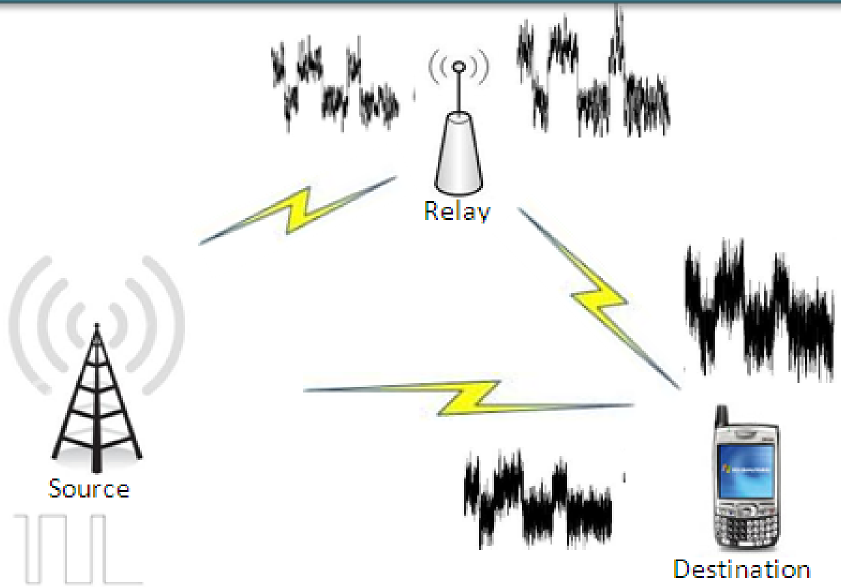
Relaying Techniques

- Amplify-and-Forward (AF)

- Decode-and-Forward (DF)

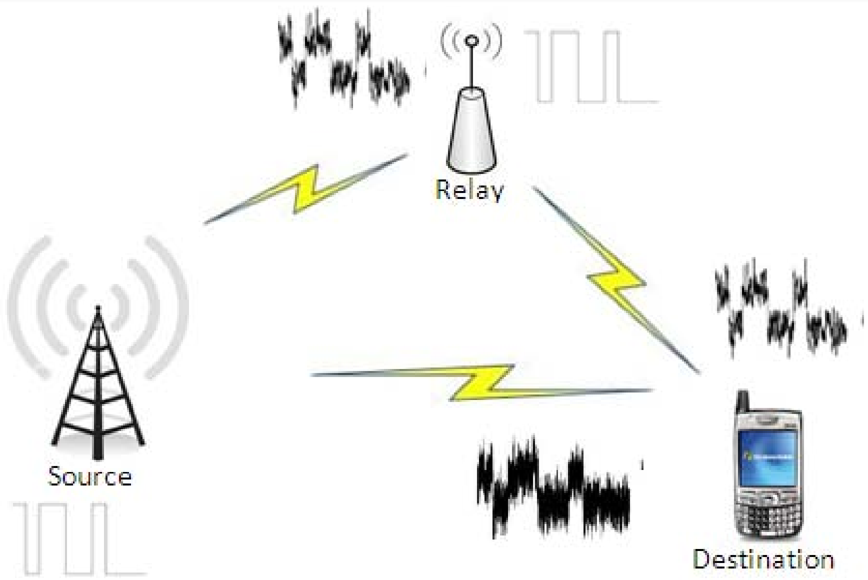
Power Consumption !
Processing Complexity !
Security !

Amplify-and-Forward (AF)



- Relay amplifies the received signals before forwarding to the destination (non-regenerative)
- **Noise and interference are also amplified and transmitted to the destination by relays**

Decode-and-Forward (DF)



- Relay receives a packet, decodes it, re-encodes it, and retransmits it (regenerative)
- **Unlike AF, when relay decodes correctly, retransmitted packet is noise and interference-free**

"AF" Cooperative Networks

The instantaneous end-to-end SNR of a "Multihop" path:

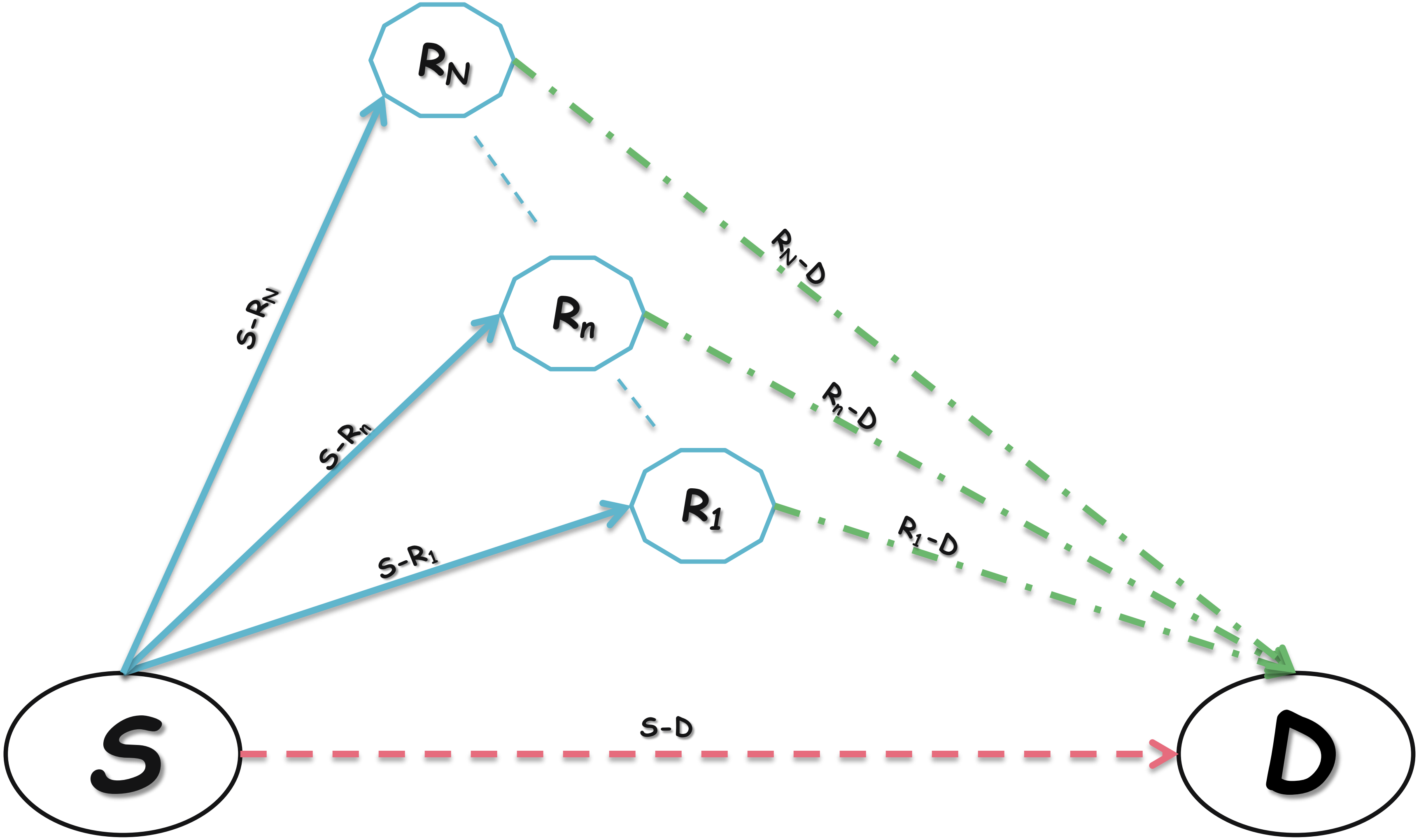
$$\gamma_t = \left[\prod_{n=1}^{N} \left(1 + \frac{1}{\gamma_n} \right) - 1 \right]^{-1} \quad \text{where,} \quad \gamma_n = \frac{P_{n-1}}{N_{0\,n}} \left| \alpha_n \right|^2$$

The instantaneous end-to-end SNR of a "Dual-hop" path:

$$\gamma_t = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}$$

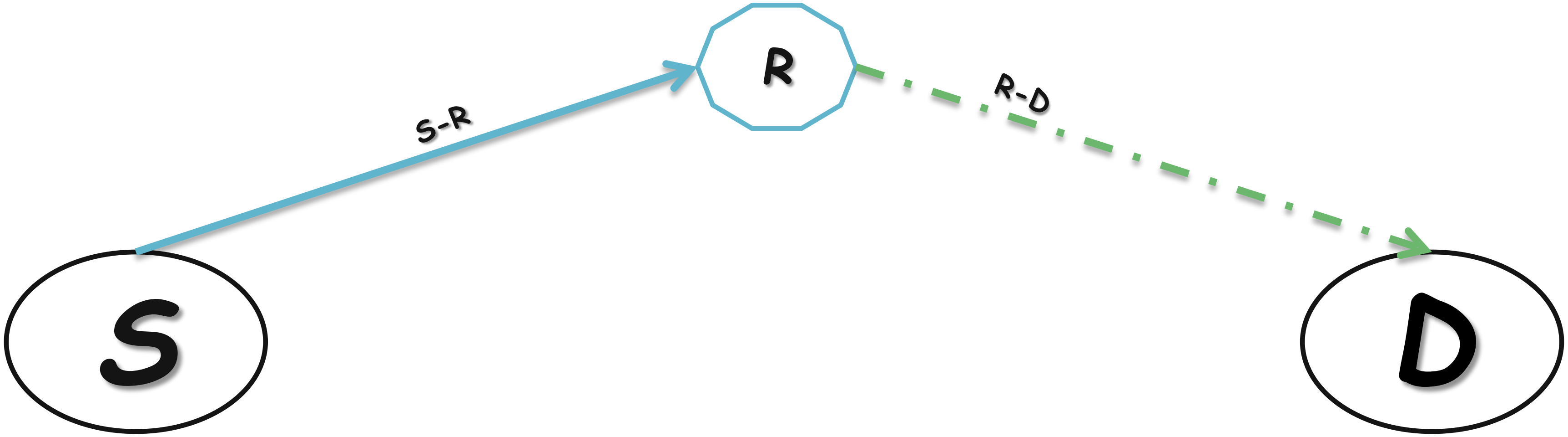
Outline

Generic System Model

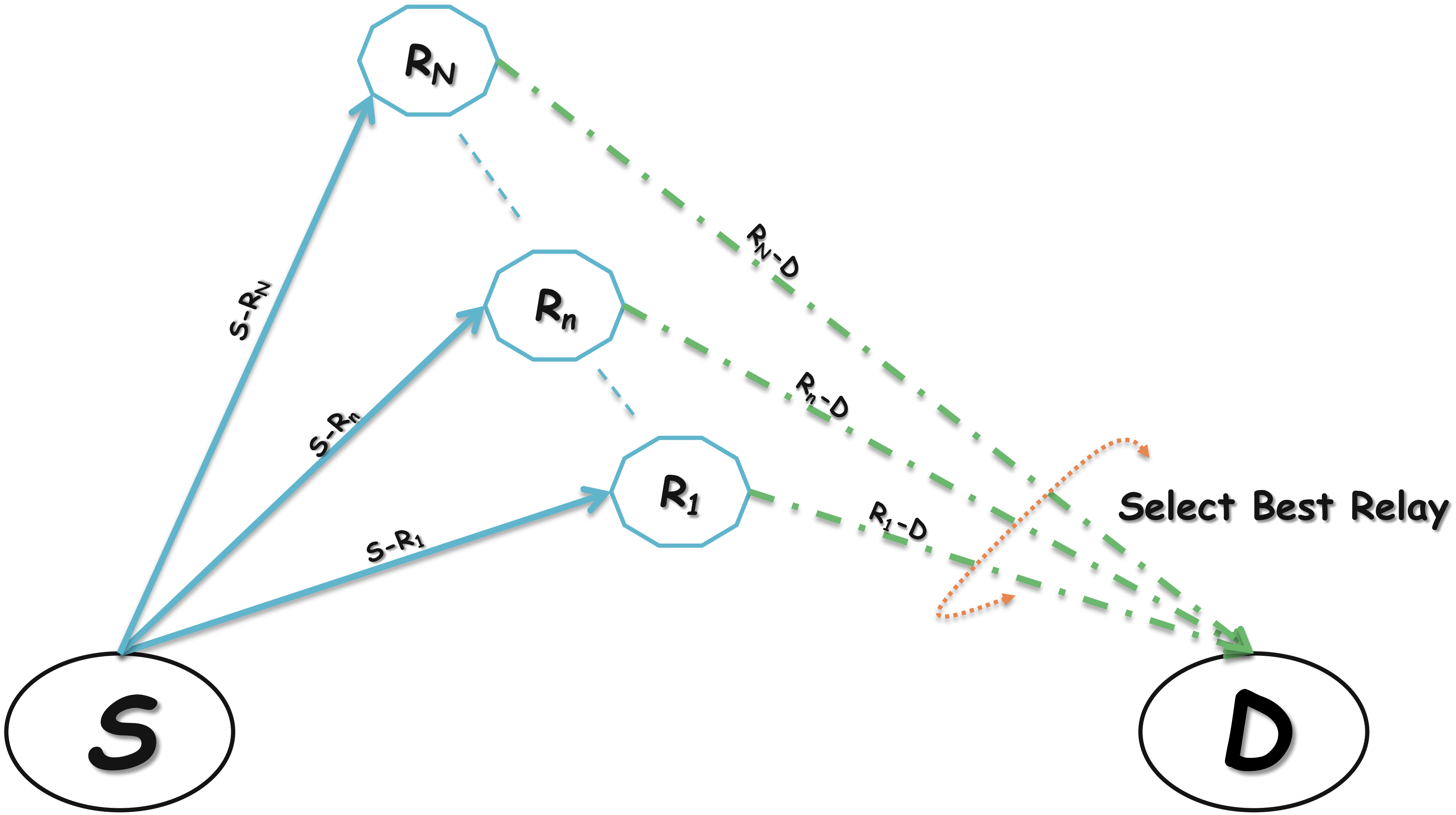


Conventional Dual-Hop Systems

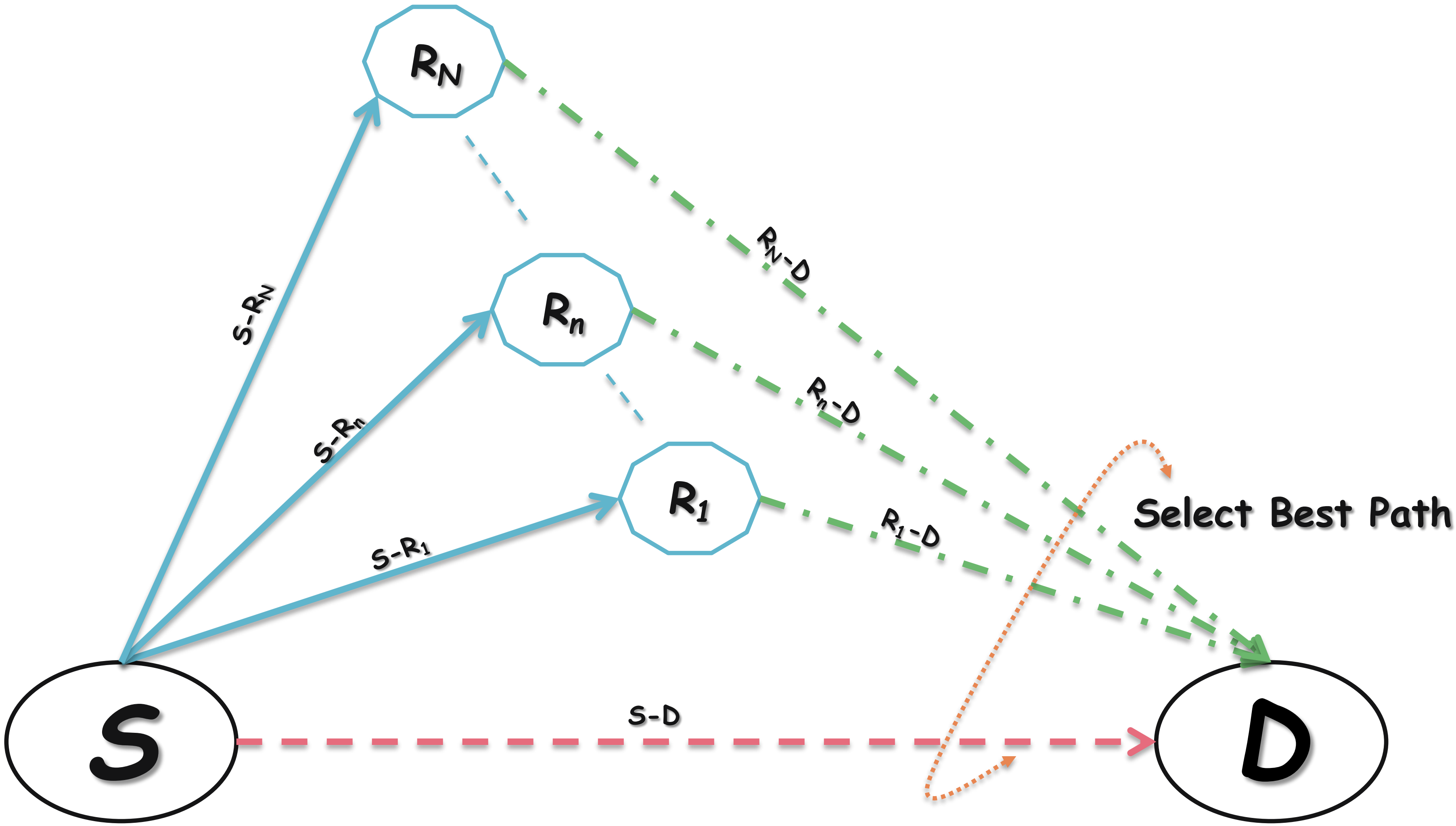
Only one user node is used to relay the radio signal between the source and the destination.



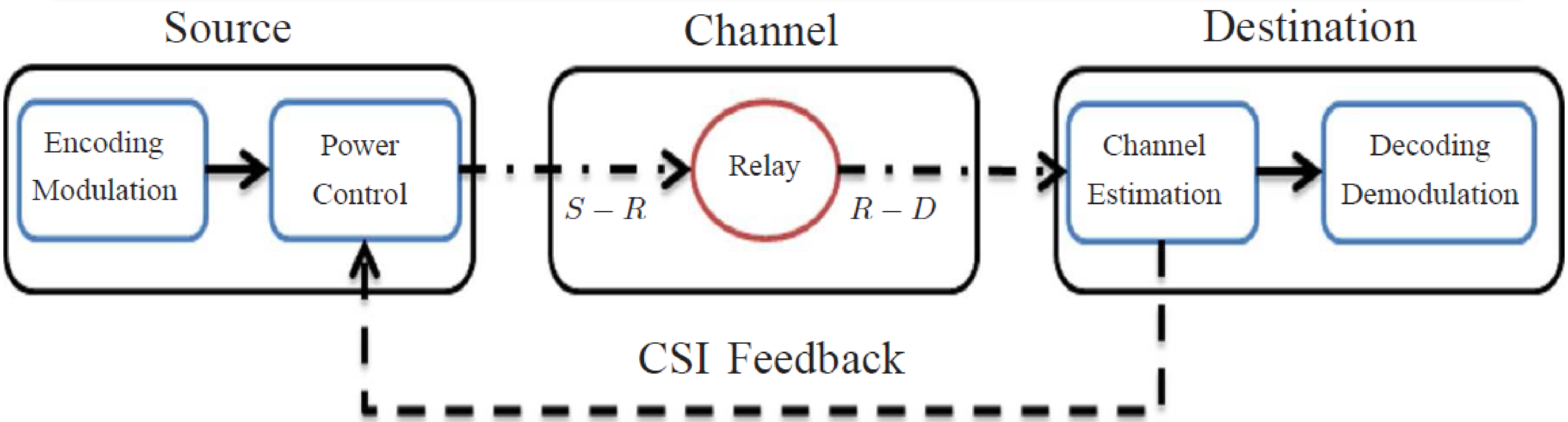
Maximum End-to-End SNR Relay Selection



Full Selection Dual-Hop Systems



With Adaptive Transmission



Optimal power and rate adaptation
Optimal rate adaptation with constant transmit power
Channel inversion with fixed rate

Problem Formulation

Necessity of having "Exact" solution for the performance metrics of dual-hop cooperative AF relaying systems.

1- Ergodic capacity.

$$\varepsilon = \frac{1}{2} E\left\{\log_2(1+\gamma_t)\right\} = \frac{1}{2}\int_0^\infty \log_2(1+\gamma_t) f_{\gamma_t}(r)dr$$

2- Outage probability.

$$P_{out} = Pr\left\{\gamma_t \leqslant \gamma_{th}\right\} = F_{\gamma_t}(\gamma_{th})$$

3- Average error probability.

$$P_s = E\left\{b\,Q\left(\sqrt{a\,\gamma_t}\right)\right\} = \int_0^\infty b\,Q\left(\sqrt{a\,\gamma_t}\right) f_{\gamma_t}(r)dr$$

Solutions in The Literature

Bounds:

- Ex1: Harmonic mean

$$\gamma_{t_1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}$$

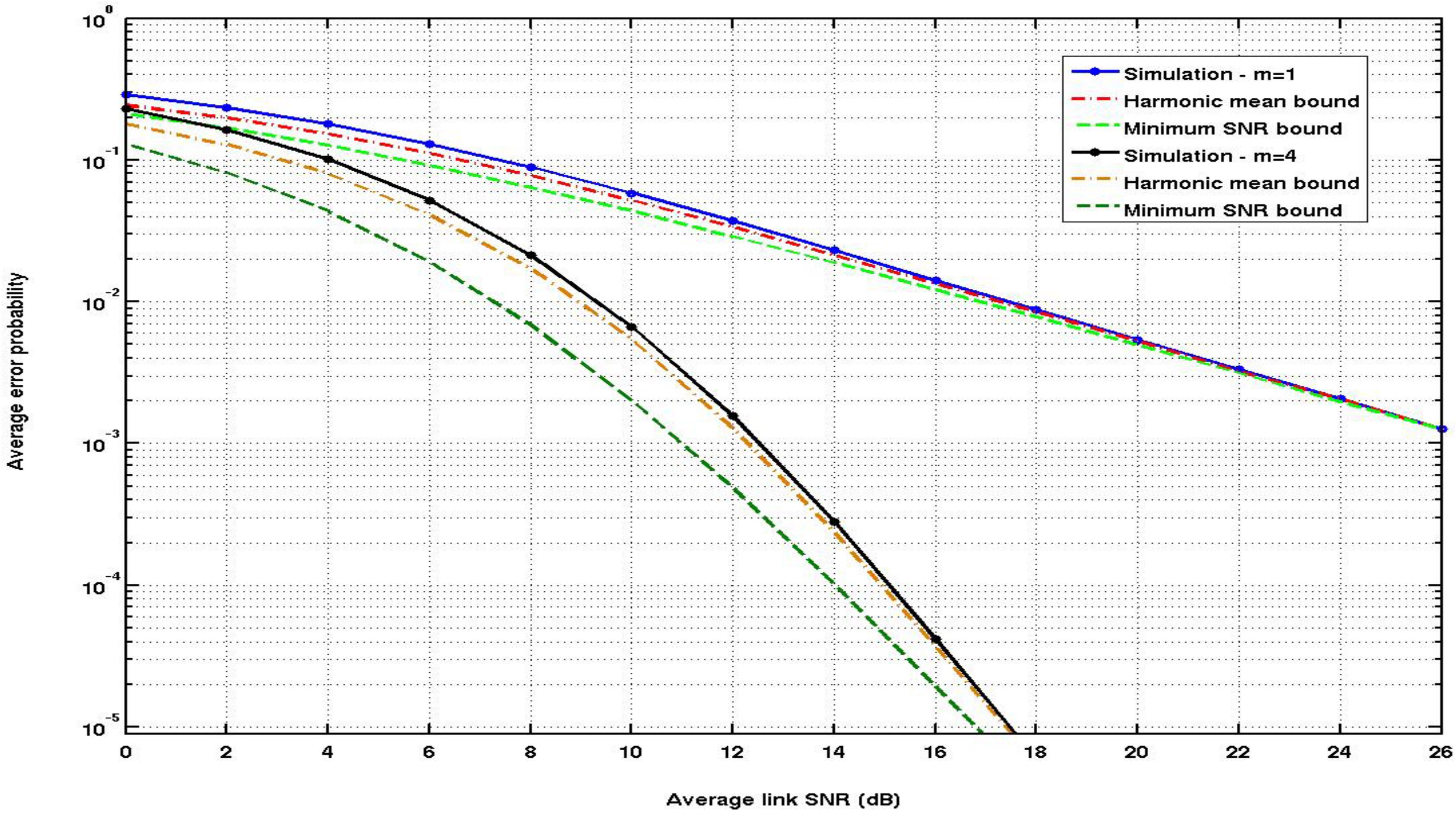
- Ex2: Minimum SNR

$$\gamma_{t_2} = \min\{\gamma_1, \gamma_2\}$$

Tightness of the Bound !
Validity for different SNR ranges !
Validity for different channel parameters !

Solutions in The Literature

❑ Bounds:



Solutions in The Literature

Approximations:

➢ Ex3: Scaling factor approximation

$$\gamma_{t_3} = \frac{c\gamma_1 c\gamma_2}{c\gamma_1 + c\gamma_2}$$

Apply a scaling factor to the harmonic mean bound to compensate for the gap between the bound and the exact results, especially for small to medium ranges of the SNR.

- Approximation (not a bound)
- More accurate
- Same computational complexity

Solutions in The Literature

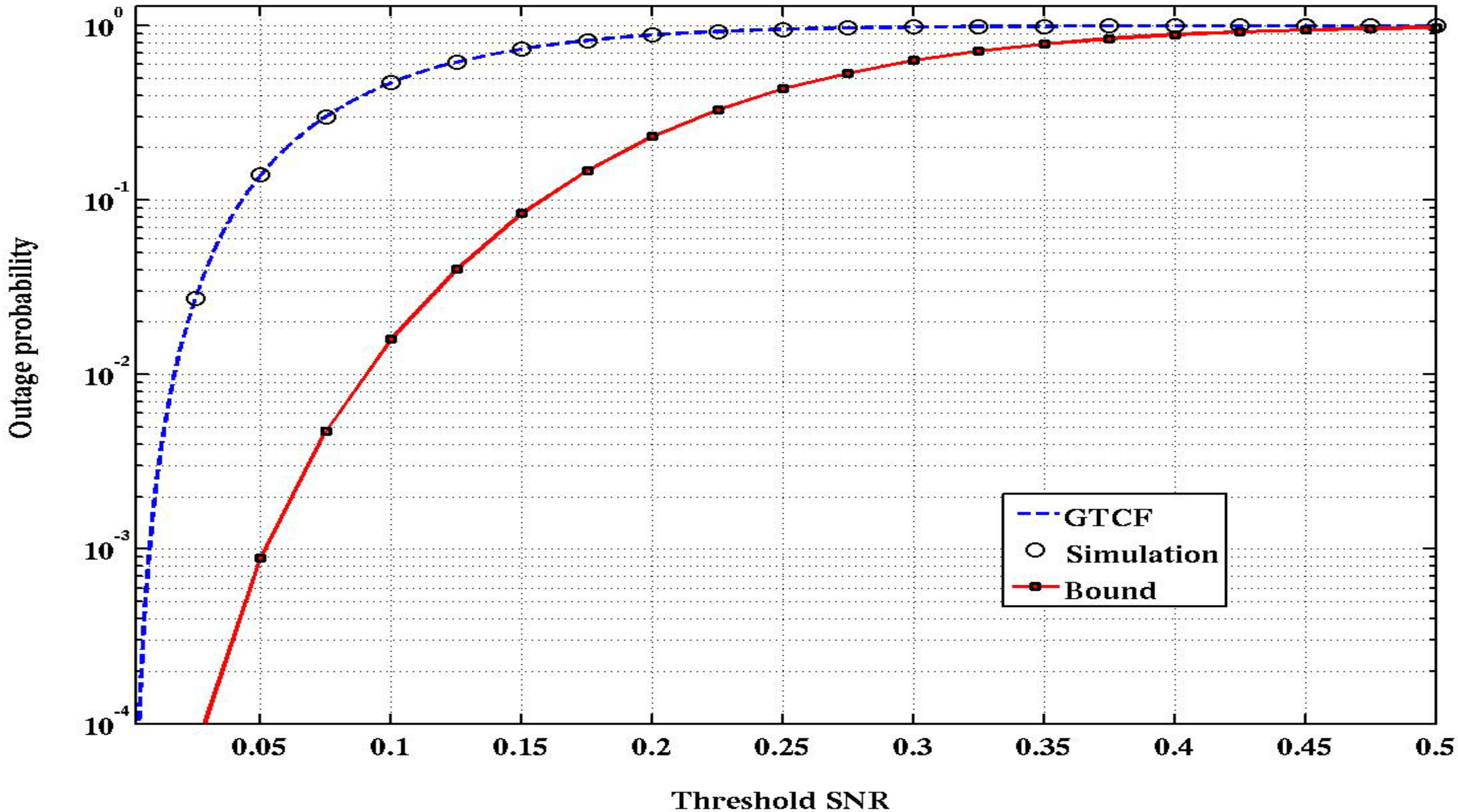
Exact:

- Ex4: GTCF (Generalized Transformed Characteristic Function)
- Ex5: M-GTCF (Modified-GTCF) ← Recently proposed

- Exact (not approximation or bound)
- Valid for any fading distribution
- Computational complexity for multihop AF systems
  → 2-fold numerical integration
- Reduced computational complexity for P_{out}
  → 1-fold numerical integration

GTCF vs Harmonic Mean Bound

Outage Probability



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Dual-Hop AF Systems: PDF

Target:

➢Obtain exact closed-form for the PDF of the end-to-end SNR.

How?

➢We start by obtaining the PDF of the instantaneous end-to-end SNR *CONDITIONED* on the second link's SNR.

$$f_{\gamma_t|\gamma_2}(\gamma_t|\gamma_2) = f\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}|\gamma_2\right)$$

➢According to a theorem on transformations of random variables, this can be simplified to:

$$f_{\gamma_t|\gamma_2}(r) = \frac{\gamma_2(\gamma_2+1)}{(\gamma_2-r)^2} f_{\gamma_1}\left(\frac{(\gamma_2+1)r}{\gamma_2-r}\right), \quad 0 \leqslant r \leqslant \gamma_2$$

Dual-Hop AF Systems: PDF

➢ The final PDF can be then obtained as:

$$f_{\gamma_t}(r) = \int_{\gamma_2=r}^{\infty} \frac{\gamma_2(\gamma_2+1)}{(\gamma_2-r)^2} f_{\gamma_1}\left(\frac{(\gamma_2+1)r}{\gamma_2-r}\right) f_{\gamma_2}(\gamma_2)\, d\gamma_2$$

✓This can be used with any arbitrary fading distributions.
✓Single-fold integral.
✓Can be solved in closed-form for the common fading distributions:

Rayleigh Fading
Nakagami-*m* Fading
Rician Fading

Dual-Hop AF Systems (Nakagami-*m*)

Independent non-identically distributed Nakagami-*m* fading links

$$f_{\gamma_t}(r) = \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} \left(\left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \left(\frac{(x+1)r}{x-r}\right)^{m_1-1} \exp\left[-\frac{1}{\overline{\gamma}_1}\left(\frac{(x+1)r}{x-r}\right)\right]\right) \times \left(\left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)} x^{m_2-1} \exp\left[-\frac{1}{\overline{\gamma}_2}x\right]\right) dx$$

which can be written as

$$f_{\gamma_t}(r) = C_g\, I_1(r; m_1-1, m_2-1, \frac{m_1}{\overline{\gamma}_1}, \frac{m_2}{\overline{\gamma}_2})$$

where,

$$C_g = \left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)}$$

and,

$$I_1(r;\alpha,\beta,\theta,\phi) = \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} \left[\frac{(x+1)\,r}{x-r}\right]^{\alpha} x^{\beta} \exp\left(-\theta\left[\frac{(x+1)\,r}{x-r}\right]\right) \exp\left(-\phi\, x\right) dx$$

Dual-Hop AF Systems (Nakagami-*m*)

Independent non-identically distributed Nakagami-*m* fading links

We noticed that the integral, $I_1(r;\alpha,\beta,\theta,\phi)$, evolves in many problems, so we solve it explicitly.

Using a change of variables, and using the binomial expansion, assuming integer values for α and β, we can obtain a closed-form expression for $I_1(r;\alpha,\beta,\theta,\phi)$ as:

$$I_1(r;\alpha,\beta,\theta,\phi) = 2\exp\left[-\left(\theta+\phi\right)r\right] r^{(\alpha+\beta)} \sum_{k=0}^{\alpha}\sum_{j=0}^{\beta} {}^{\alpha}C_k {}^{\beta}C_j \left(\frac{\theta}{\phi}\right)^{\frac{j-k}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k}{2}}$$
$$\times\left[\sqrt{\frac{\theta}{\phi}r(r+1)}\,K_{j-k+1}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right) + (2r+1)\,K_{j-k}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right)\right.$$
$$\left. + \sqrt{\frac{\phi}{\theta}r(r+1)}\,K_{j-k-1}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right)\right]$$

Dual-Hop AF Systems (Nakagami-*m*)

Independent non-identically distributed Nakagami-*m* fading links

Then, the exact PDF of the end-to-end SNR in the case of (*i.ni.d.*) Nakagami-*m* fading channels is given by:

$$
\begin{aligned}
f_{\gamma_t}(r) = {} & 2\left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1}\frac{1}{\Gamma(m_1)}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2}\frac{1}{\Gamma(m_2)}\exp\left[-\left(\frac{m_1}{\overline{\gamma}_1}+\frac{m_2}{\overline{\gamma}_2}\right)r\right]r^{m_1+m_2-2} \\
& \times\sum_{k=0}^{m_1-1}\sum_{j=0}^{m_2-1}{}^{m_1-1}C_k\,{}^{m_2-1}C_j\left(\frac{m_1\overline{\gamma}_2}{m_2\overline{\gamma}_1}\right)^{\frac{j-k}{2}}\left(\frac{r+1}{r}\right)^{\frac{j+k}{2}} \\
& \times\left[\sqrt{\frac{m_1\overline{\gamma}_2}{m_2\overline{\gamma}_1}}\,r(r+1)\,K_{j-k+1}\left(2\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2}r(r+1)}\right)\right. \\
& +(2r+1)\,K_{j-k}\left(2\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2}r(r+1)}\right) \\
& \left.+\sqrt{\frac{m_2\overline{\gamma}_1}{m_1\overline{\gamma}_2}}\,r(r+1)\,K_{j-k-1}\left(2\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2}r(r+1)}\right)\right]
\end{aligned}
$$

Dual-Hop AF Systems (Rician)

Independent non-identically distributed Rician fading links

$$f_{\gamma_t}(r) = \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} \left(\frac{1+K_2}{\overline{\gamma}_2} e^{-K_2} \exp\left[-\frac{1+K_2}{\overline{\gamma}_2} x \right] I_0\left(2\sqrt{\frac{K_2(1+K_2)}{\overline{\gamma}_2} x} \right) \right)$$
$$\times \left(\frac{1+K_1}{\overline{\gamma}_1} e^{-K_1} \exp\left[-\frac{1+K_1}{\overline{\gamma}_1} \left(\frac{(x+1)r}{x-r} \right) \right] I_0\left(2\sqrt{\frac{K_1(1+K_1)}{\overline{\gamma}_1} \left(\frac{(x+1)r}{x-r} \right)} \right) \right) dx$$

To proceed, we use the infinite series representation of the Bessel function;

$$I_0\left(2\sqrt{\frac{K_i(1+K_i)}{\overline{\gamma}_i} x} \right) = \sum_{n=0}^{\infty} a_i(n) x^n$$

where,

$$a_i(n) = \frac{1}{(n!)^2} \left(\frac{K_i(1+K_i)}{\overline{\gamma}_i} \right)^n$$

Dual-Hop AF Systems (Rician)

Independent non-identically distributed Rician fading links

Substituting into the PDF expression, we obtain:

$$f_{\gamma_t}(r) = \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} \left(\frac{1+K_2}{\overline{\gamma}_2} e^{-K_2} \exp\left[-\frac{1+K_2}{\overline{\gamma}_2} x \right] \sum_{m=0}^{\infty} a_2(m) x^m \right)$$
$$\times \left(\frac{1+K_1}{\overline{\gamma}_1} e^{-K_1} \exp\left[-\frac{1+K_1}{\overline{\gamma}_1} \left(\frac{(x+1)r}{x-r} \right) \right] \sum_{n=0}^{\infty} a_1(n) \left(\frac{(x+1)r}{x-r} \right)^n \right) dx$$

Interchanging the order of the summation and integration, we obtain:

$$f_{\gamma_t}(r) = \left(\frac{1+K_1}{\overline{\gamma}_1} \right) \left(\frac{1+K_2}{\overline{\gamma}_2} \right) e^{-(K_1+K_2)} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_1(n) a_2(m) \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2}$$
$$\times \left(\frac{(x+1)r}{x-r} \right)^n \exp\left[-\frac{1+K_1}{\overline{\gamma}_1} \left(\frac{(x+1)r}{x-r} \right) \right] x^m \exp\left[-\frac{1+K_2}{\overline{\gamma}_2} x \right] dx$$

Dual-Hop AF Systems (Rician)

Independent non-identically distributed Rician fading links

Recognizing the integral expression as $I_1(r; n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2})$, we obtain the exact PDF of the end-to-end SNR in the case of (*i.ni.d.*) Rician fading channels as:

$$f_{\gamma_t}(r) = 2\left(\frac{1+K_1}{\overline{\gamma}_1}\right)\left(\frac{1+K_2}{\overline{\gamma}_2}\right)e^{-(K_1+K_2)}\exp\left[-\left(\frac{1+K_1}{\overline{\gamma}_1}+\frac{1+K_2}{\overline{\gamma}_2}\right)r\right]$$
$$\times\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}a_1(n)a_2(m)\,r^{n+m}\sum_{k=0}^{n}\sum_{j=0}^{m}{}^{n}C_k\,{}^{m}C_j\left(\frac{(1+K_1)\overline{\gamma}_2}{(1+K_2)\overline{\gamma}_1}\right)^{\frac{j-k}{2}}\left(\frac{r+1}{r}\right)^{\frac{j+k}{2}}$$
$$\times\left[\sqrt{\frac{(1+K_1)\overline{\gamma}_2}{(1+K_2)\overline{\gamma}_1}r(r+1)}\,K_{j-k+1}\left(2\sqrt{\frac{(1+K_1)(1+K_2)}{\overline{\gamma}_1\overline{\gamma}_2}r(r+1)}\right)\right.$$
$$+(2r+1)\,K_{j-k}\left(2\sqrt{\frac{(1+K_1)(1+K_2)}{\overline{\gamma}_1\overline{\gamma}_2}r(r+1)}\right)$$
$$\left.+\sqrt{\frac{(1+K_2)\overline{\gamma}_1}{(1+K_1)\overline{\gamma}_2}r(r+1)}\,K_{j-k-1}\left(2\sqrt{\frac{(1+K_1)(1+K_2)}{\overline{\gamma}_1\overline{\gamma}_2}r(r+1)}\right)\right]$$

Dual-Hop AF Systems (Rician)

Independent non-identically distributed Rician fading links

➢Note that although the previous expression involves double nested infinite summations, the summands decay exponentially, or slightly faster, because of the $\frac{1}{(n!)^2}$ and $\frac{1}{(m!)^2}$ factors.

➢Stirling's approximation specifies that $n!$ grows as $e^{n \ln n}$, and hence $\frac{1}{(n!)^2}$ decays as $e^{-2n \ln n}$.

➢Truncated summation with a finite number of terms will achieve a required accuracy.

Dual-Hop AF Systems (Mixed)

Independent mixed Nakagami-*m* and Rician fading links

$$f_{\gamma_t}(r) = \left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \frac{1+K_2}{\overline{\gamma}_2} e^{-K_2}$$
$$\times \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} \left(\frac{(x+1)r}{x-r}\right)^{m_1-1} \exp\left[-\frac{m_1}{\overline{\gamma}_1}\left(\frac{(x+1)r}{x-r}\right)\right]$$
$$\times \exp\left[-\frac{1+K_2}{\overline{\gamma}_2}x\right] I_0\left(2\sqrt{\frac{K_2(1+K_2)}{\overline{\gamma}_2}x}\right) dx.$$

To proceed, we use the infinite series representation of the Bessel function, the binomial series expansion and follow similar procedure as done before.

Dual-Hop AF Systems (Mixed)

Independent mixed Nakagami-*m* and Rician fading links

The exact PDF of the end-to-end SNR in the case of mixed Nakgami-*m* and Rician fading links is given as:

$$f_{\gamma_t}(r) = C_f \sum_{n=0}^{\infty} a_2(n) I_1\left(r; m_1 - 1, n, \frac{m_1}{\overline{\gamma}_1}, \delta_2\right)$$

where,

$$C_f = \left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \delta_2 e^{-K_2}$$

and,

$$\delta_2 = \frac{1+K_2}{\overline{\gamma}_2}$$

Dual-Hop AF Systems: CDF

Target:

➢Obtain exact closed-form for the CDF of the end-to-end SNR.

How?

➢We use the expression obtained previously for the conditional PDF of the end-to-end SNR, to obtain:

$$P_{out} = \int_{r=0}^{\gamma_{th}} \int_{\gamma_2=r}^{\infty} f_{\gamma_t|\gamma_2}(r) f_{\gamma_2}(\gamma_2) d\gamma_2 \, dr$$

➢Changing the order of the integration, we obtain:

$$P_{out} = \int_{\gamma_2=0}^{\gamma_{th}} \left(\int_{r=0}^{\gamma_2} f_{\gamma_t|\gamma_2}(r) \, dr \right) f_{\gamma_2}(\gamma_2) \, d\gamma_2 + \int_{\gamma_2=\gamma_{th}}^{\infty} \left(\int_{r=0}^{\gamma_{th}} f_{\gamma_t|\gamma_2}(r) \, dr \right) f_{\gamma_2}(\gamma_2) \, d\gamma_2$$

Dual-Hop AF Systems: CDF

$$P_{out} = \int_{\gamma_2=0}^{\gamma_{th}} \left(\int_{r=0}^{\gamma_2} f_{\gamma_t|\gamma_2}(r)\, dr \right) f_{\gamma_2}(\gamma_2)\, d\gamma_2 + \int_{\gamma_2=\gamma_{th}}^{\infty} \left(\int_{r=0}^{\gamma_{th}} f_{\gamma_t|\gamma_2}(r)\, dr \right) f_{\gamma_2}(\gamma_2)\, d\gamma_2$$

Since $0 < r < \gamma_2$, this integral goes to "1"

And hence, this term is equal to $F_{\gamma_2}(\gamma_{th})$

Using a change of variables, this integral can be written as:

$$\int_{r=0}^{\gamma_{th}} f_{\gamma_t|\gamma_2}(r)\, dr = \int_{\gamma_1=0}^{\frac{(\gamma_2+1)\gamma_{th}}{\gamma_2-\gamma_{th}}} f_{\gamma_1}(\gamma_1)\, d\gamma_1 = F_{\gamma_1}\left(\frac{(\gamma_2+1)\gamma_{th}}{\gamma_2-\gamma_{th}} \right)$$

Then the outage probability (CDF) can be written as:

$$P_{out} = F_{\gamma_t}(\gamma_{th}) = F_{\gamma_2}(\gamma_{th}) + \int_{\gamma_2=\gamma_{th}}^{\infty} F_{\gamma_1}\left(\frac{(\gamma_2+1)\gamma_{th}}{\gamma_2-\gamma_{th}} \right) f_{\gamma_2}(\gamma_2)\, d\gamma_2$$

Dual-Hop AF Systems (Nakagami-*m*)

Independent non-identically distributed Nakagami-*m* fading links

$$F_{\gamma_t}(\gamma_{th}) = F_{\gamma_2}(\gamma_{th}) + \int_{\gamma_2=\gamma_{th}}^{\infty} \left(\frac{\Gamma_{inc}\left(m_1, \frac{m_1}{\overline{\gamma}_1}\left(\frac{(\gamma_2+1)\gamma_{th}}{\gamma_2-\gamma_{th}}\right)\right)}{\Gamma(m_1)} \right) f_{\gamma_2}(\gamma_2)\, d\gamma_2$$

We use the finite series representation of the lower incomplete gamma function,

$$\frac{\Gamma_{inc}\left(m_1, \frac{m_1}{\overline{\gamma}_1} r\right)}{\Gamma(m_1)} = 1 - \exp\left[-\frac{m_1}{\overline{\gamma}_1} r\right] \sum_{n=0}^{m_1-1} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1} r\right)^n$$

Dual-Hop AF Systems (Nakagami-*m*)

Independent non-identically distributed Nakagami-*m* fading links

The CDF can be then written as:

$$F_{\gamma_t}(\gamma_{th}) = 1 - \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)} \sum_{n=0}^{m_1-1} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{n}$$
$$\times \int_{x=\gamma_{th}}^{\infty} \left(\frac{(x+1)\gamma_{th}}{x-\gamma_{th}}\right)^{n} \exp\left[-\frac{m_1}{\overline{\gamma}_1}\left(\frac{(x+1)\gamma_{th}}{x-\gamma_{th}}\right)\right] \left(x^{m_2-1}\exp\left[-\frac{m_2}{\overline{\gamma}_2}x\right]\right) dx$$

Again, the above integral is recognized in many cases, so it is explicitly solved as:

$$I_2(r;\alpha,\beta,\theta,\phi) = 2\exp\left[-(\theta+\phi)\,r\right] r^{(\alpha+\beta+1)} \sum_{k=0}^{\alpha}\sum_{j=0}^{\beta} {}^{\alpha}C_k\, {}^{\beta}C_j$$
$$\times \left(\frac{\theta}{\phi}\right)^{\frac{j-k+1}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k+1}{2}} K_{j-k+1}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right)$$

Dual-Hop AF Systems (Nakagami-*m*)

Independent non-identically distributed Nakagami-*m* fading links

The CDF of the end-to-end SNR in the case of (*i.ni.d.*) Nakagami-*m* fading channels is obtained as:

$$F_{\gamma_t}(\gamma_{th}) = 1 - 2\left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)} \exp\left[-\left(\frac{m_1}{\overline{\gamma}_1} + \frac{m_2}{\overline{\gamma}_2}\right)\gamma_{th}\right] \sum_{n=0}^{m_1-1} \frac{1}{n!}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{n} {\gamma_{th}}^{n+m_2}$$
$$\times \sum_{k=0}^{n} \sum_{j=0}^{m_2-1} {}^{n}C_k \, {}^{m_2-1}C_j \times \left(\frac{m_1\overline{\gamma}_2}{m_2\overline{\gamma}_1}\right)^{\frac{j-k+1}{2}} \left(\frac{\gamma_{th}+1}{\gamma_{th}}\right)^{\frac{j+k+1}{2}}$$
$$\times K_{j-k+1}\left(2\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2}\gamma_{th}(\gamma_{th}+1)}\right)$$

Dual-Hop AF Systems (Rician)

Independent non-identically distributed Rician fading links

The CDF of the end-to-end SNR in the case of (*i.ni.d.*) Rician fading channels is obtained as:

$$F_{\gamma_t}(\gamma_{th}) = 1 - \exp\left[-(K_1 + K_2)\right] \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{{K_1}^{l+n}\, {K_2}^{m}}{(l+n)!\, n!\, (m!)^2}$$
$$\times \left(\frac{1+K_1}{\overline{\gamma}_1}\right)^{n} \left(\frac{1+K_2}{\overline{\gamma}_2}\right)^{m+1}$$
$$\times \sum_{k=0}^{n} \sum_{j=0}^{m} {}^{n}C_k \, {}^{m}C_j \times \left(\frac{(1+K_1)\,\overline{\gamma}_2}{(1+K_2)\,\overline{\gamma}_1}\right)^{\frac{j-k+1}{2}} \left(\frac{\gamma_{th}+1}{\gamma_{th}}\right)^{\frac{j+k+1}{2}}$$
$$\times K_{j-k+1}\left(2\sqrt{\frac{(1+K_1)(1+K_2)}{\overline{\gamma}_1 \overline{\gamma}_2}\gamma_{th}(\gamma_{th}+1)}\right)$$

Dual-Hop AF Systems (Mixed)

Independent mixed Nakagami-*m* and Rician fading links

The CDF of the end-to-end SNR in the case of mixed Nakagami-*m* and Rician fading channels is obtained as:

$$F_{\gamma_t}(r) = 1 - \sum_{j=0}^{m_1-1} \sum_{n=0}^{\infty} C_F(j) a_2(n) I_2\left(r; j, n, \frac{m_1}{\overline{\gamma}_1}, \delta_2\right)$$

where,

$$C_F(j) = \left(\frac{m_1}{\overline{\gamma}_1}\right)^j \frac{1}{j!} \delta_2 e^{-K_2}$$

and,

$$\delta_2 = \frac{1+K_2}{\overline{\gamma}_2} \qquad a_2(n) = \frac{1}{(n!)^2} \left(\frac{K_2(1+K_2)}{\overline{\gamma}_2}\right)^n$$

Dual-Hop AF Systems: Performance

- Outage probability (the CDF of the end-to-end SNR) is obtained in closed-form for the common channel fading distributions.

- The average error probability and ergodic capacity are obtained using a single-fold numerical integration.

Numerical Results: Assumptions

❖Uniform power allocation of the total available power, *P*.

❖Equal noise powers, N_0, at all the nodes.

❖ Nodes are located at equal distances from each other.

❖Friss propagation model; $\overline{\gamma} = N^{\delta-1}\frac{P}{N_0}$

❖BPSK

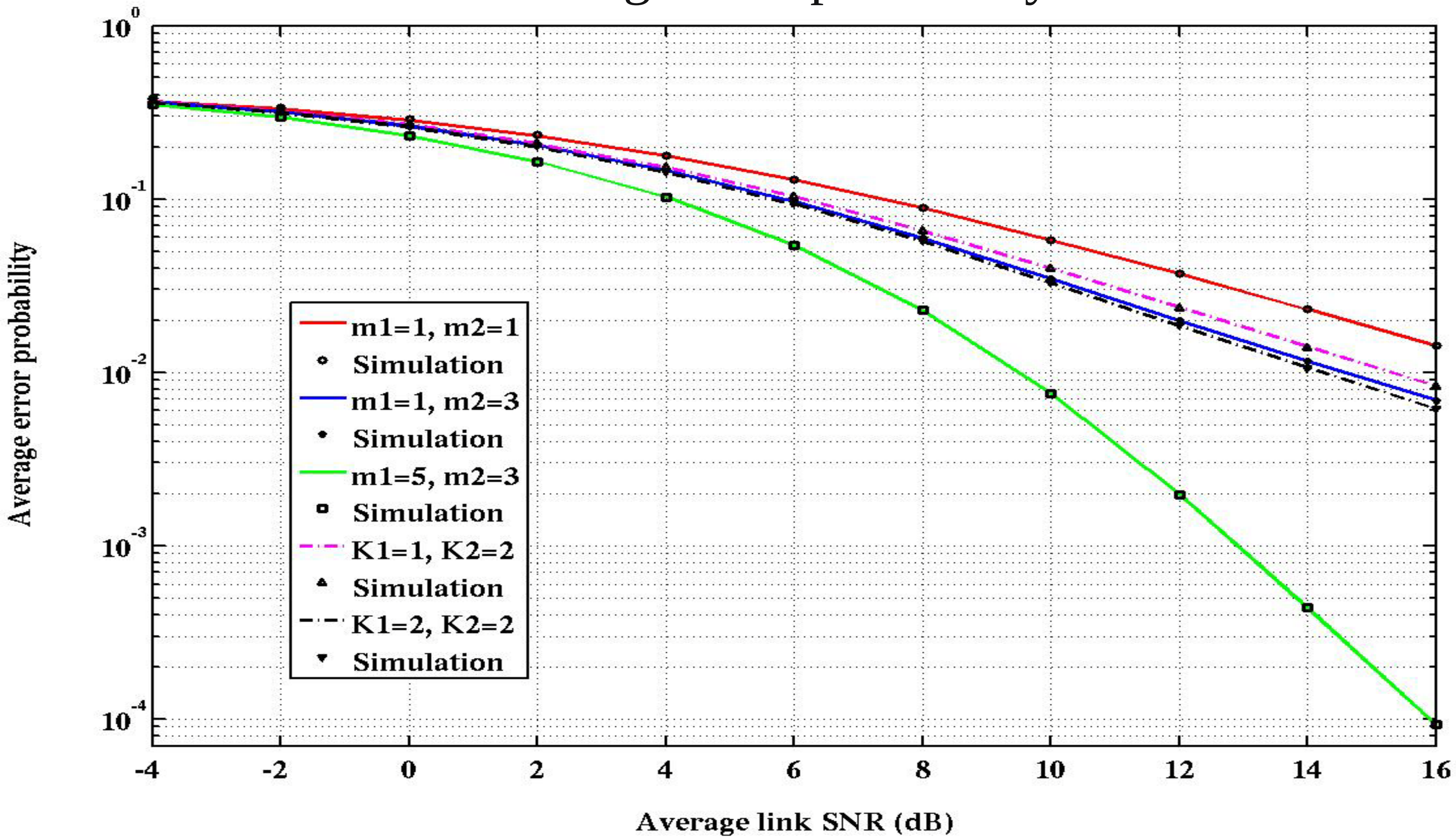
Numerical Results: Example set #1

Dual-hop AF system

- i.i.d. Rayleigh Links: $m=1$
- i.ni.d. Nakagami-m links: $m_1=1$, $m_2=3$
- i.ni.d. Nakagami-m links: $m_1=5$, $m_2=3$
- i.ni.d. Rician links: $K_1=1$, $K_2=2$
- i.i.d. Rician links: $K_1=2$, $K_2=2$

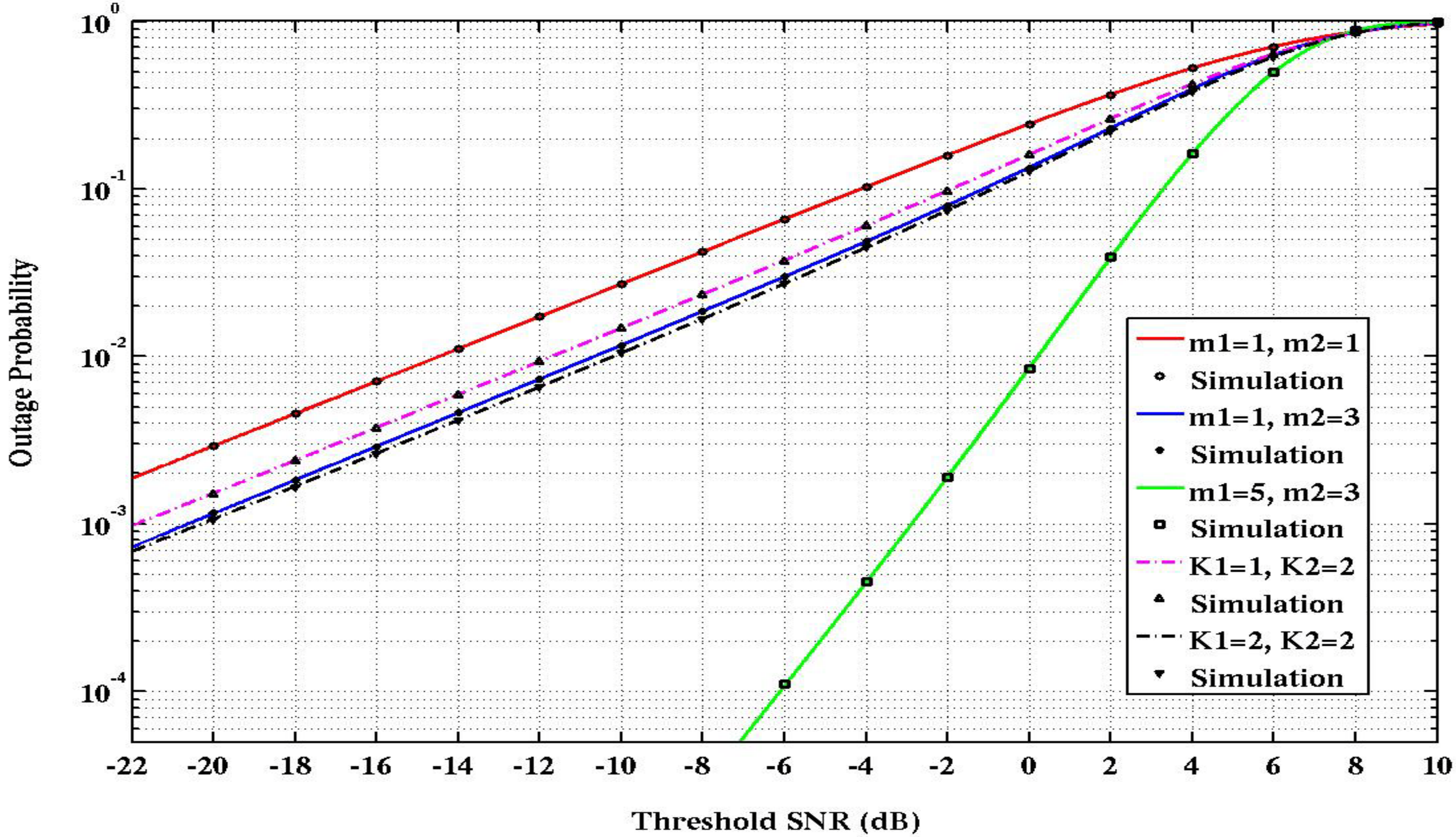
Numerical Results: Example set #1

Average error probability

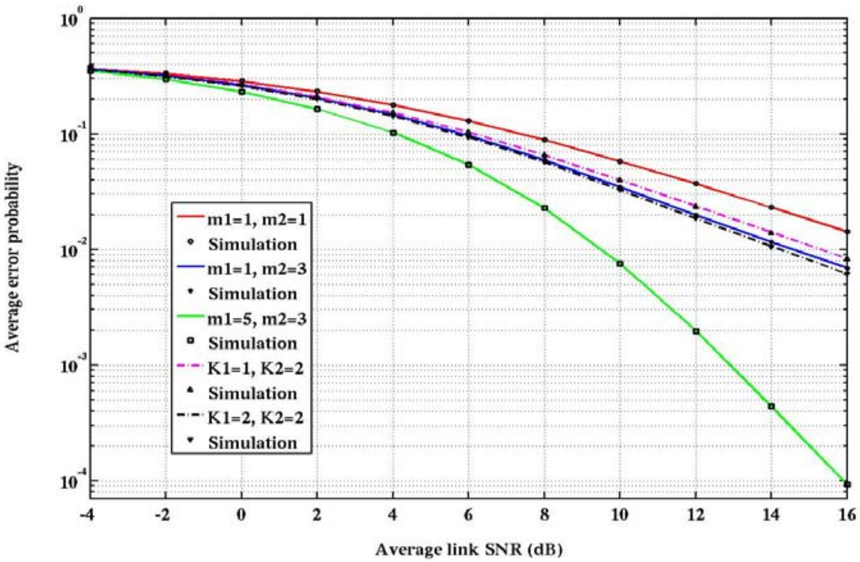


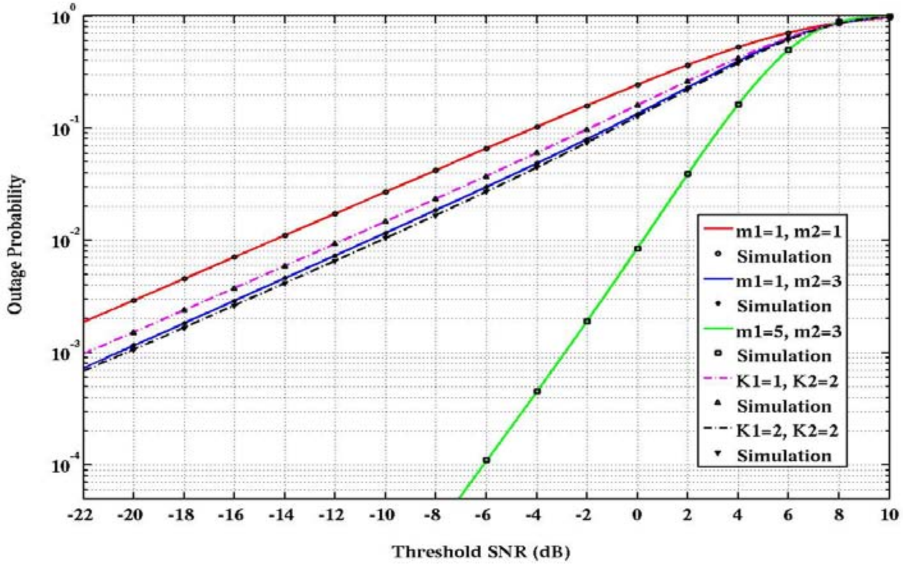
Numerical Results: Example set #1

Outage probability



Observations and Comments





1. Precise agreement with simulation results.
2. The less severe Rician and Nakagami-*m* fading channels exhibit better performance.
3. Limiting slopes of the P_s and P_{out} curves.
   - Rician curves are parallel to the Rayleigh curves.
   - Limiting slopes of the Nakagami-*m* curves are affected by the fading parameters, *m*.

Numerical Results: Example set #2

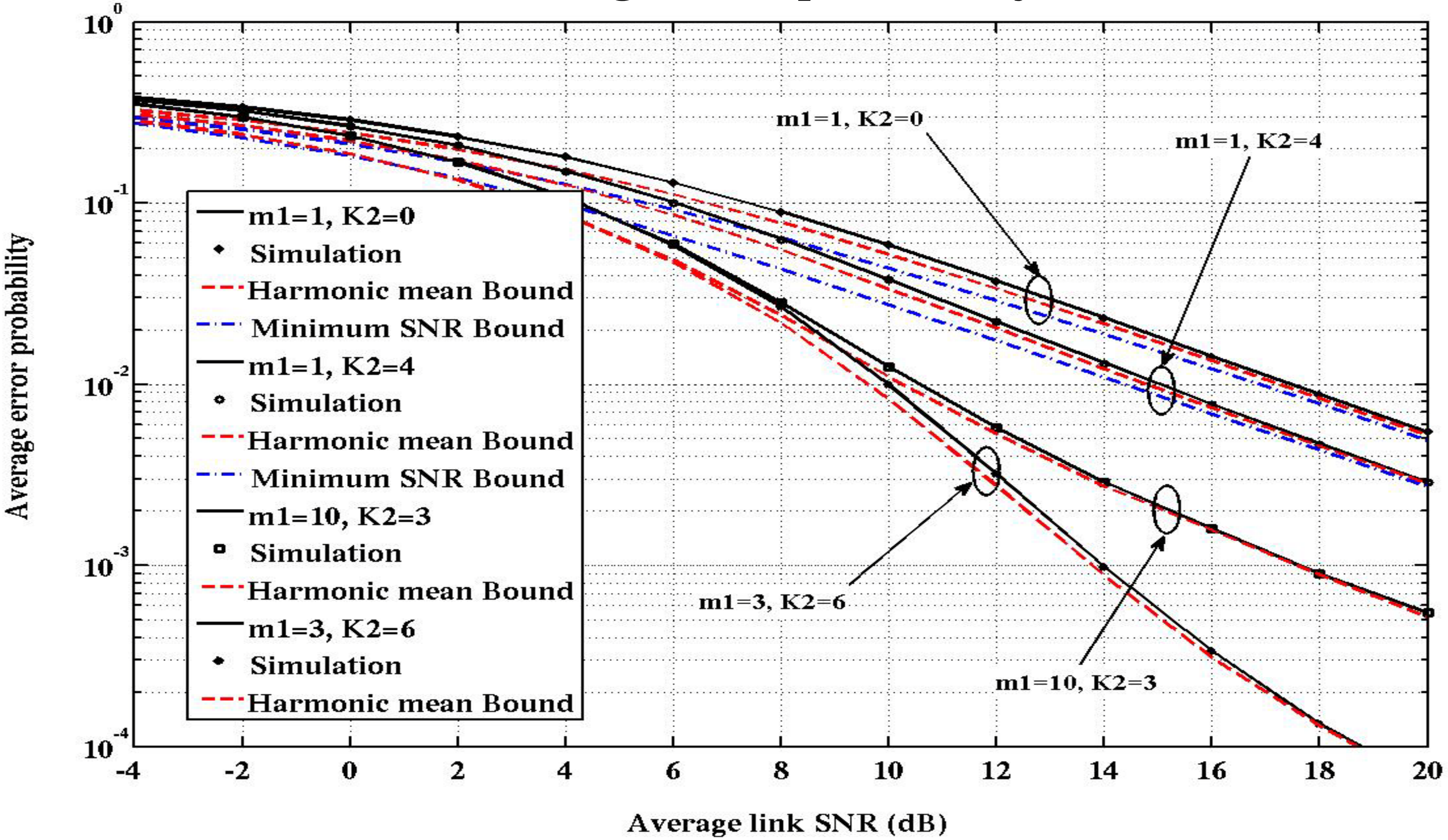
Dual-hop AF system

Mixed Nakagami-*m* and Rician Links

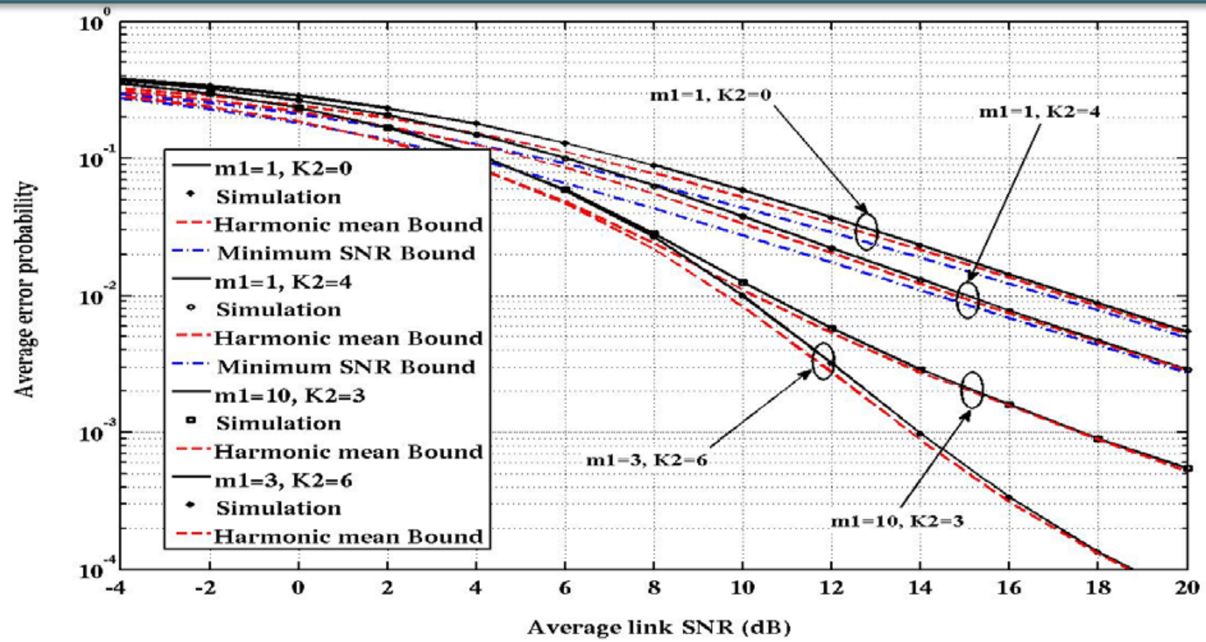
- ❑Case(1): $m_1=1,\ K_2=0$
- ❑Case(2): $m_1=1,\ K_2=4$
- ❑Case(3): $m_1=3,\ K_2=6$
- ❑Case(4): $m_1=10,\ K_2=3$

Numerical Results: Example set #2

Average error probability



Observations and Comments



1- Both the harmonic mean bound and the minimum SNR bound are not tight for small-to-moderate values of the average SNR.

2- Results obtained based on the minimum SNR bound are more loose than those obtained based on the harmonic mean bound.

→ This emphasizes the value of the exact analysis.

Numerical Results: Example set #3

Dual-hop AF system

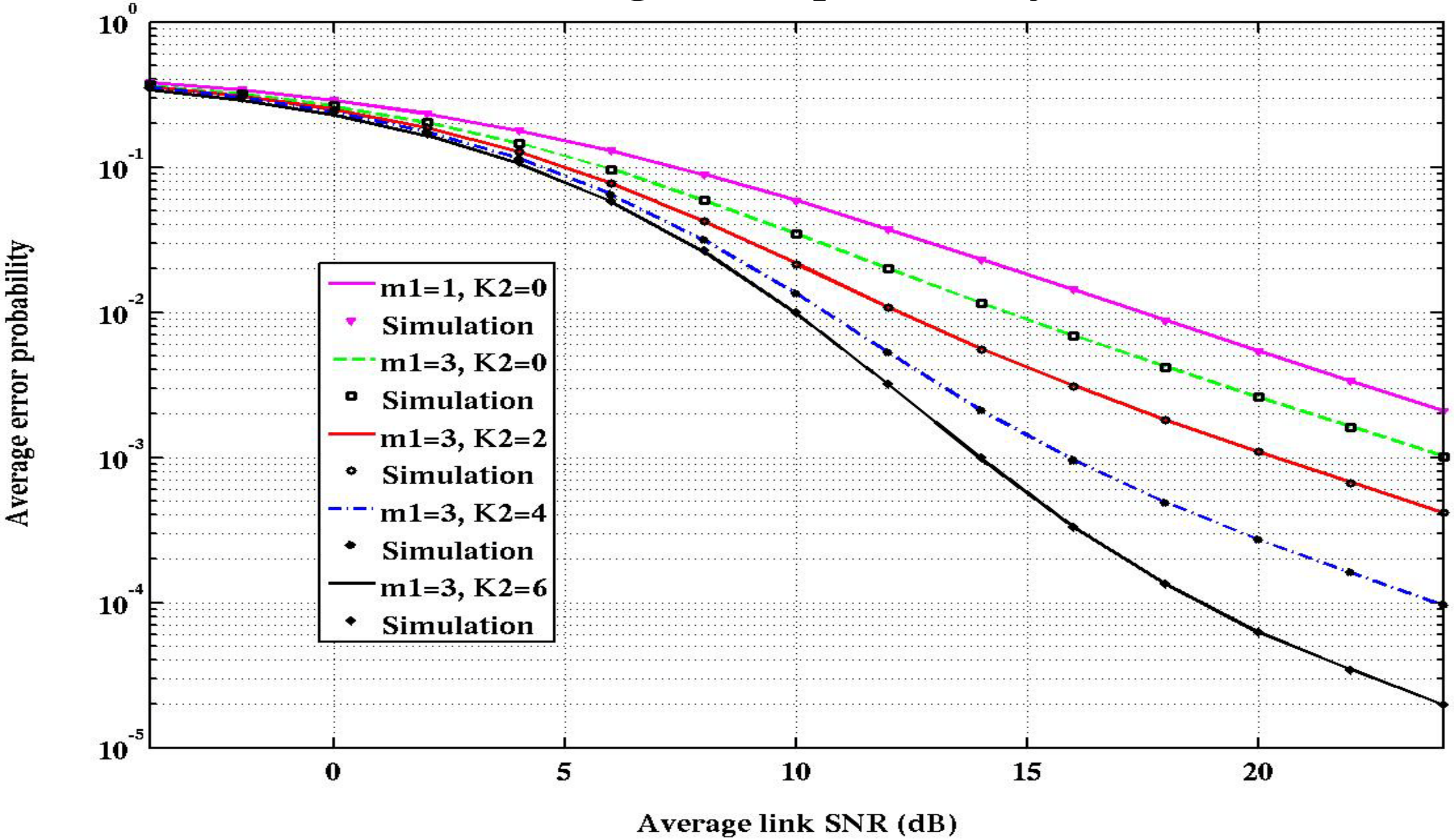
Mixed Nakagami-*m* and Rician Links

Fixed *m* and variable K

- ❑Case(1): $m_1=3, K_2=0$
- ❑Case(2): $m_1=3, K_2=2$
- ❑Case(2): $m_1=3, K_2=4$
- ❑Case(2): $m_1=1, K_2=0$
- ❑Case(2): $m_1=3, K_2=6$

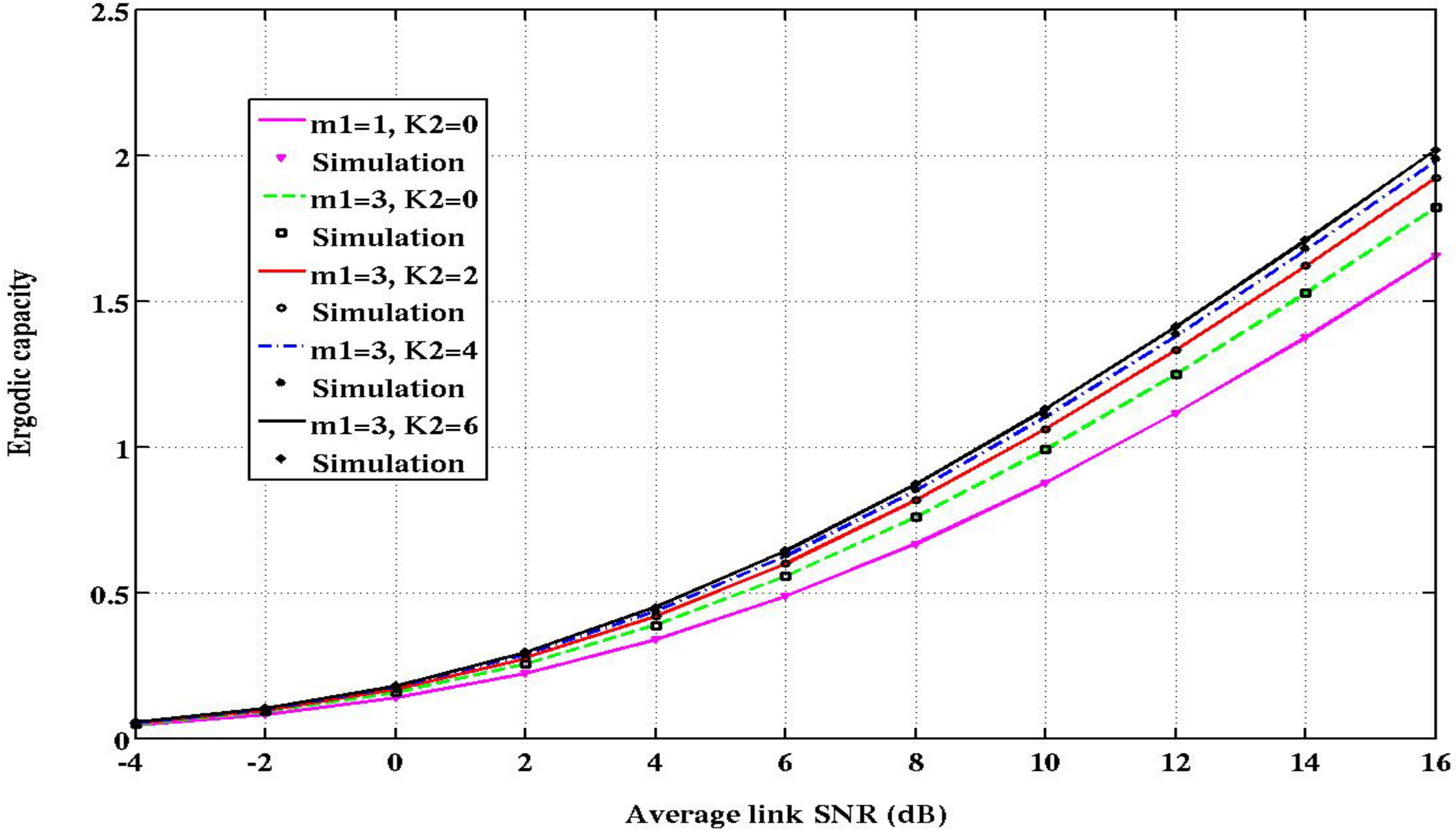
Numerical Results: Example set #3

Average error probability

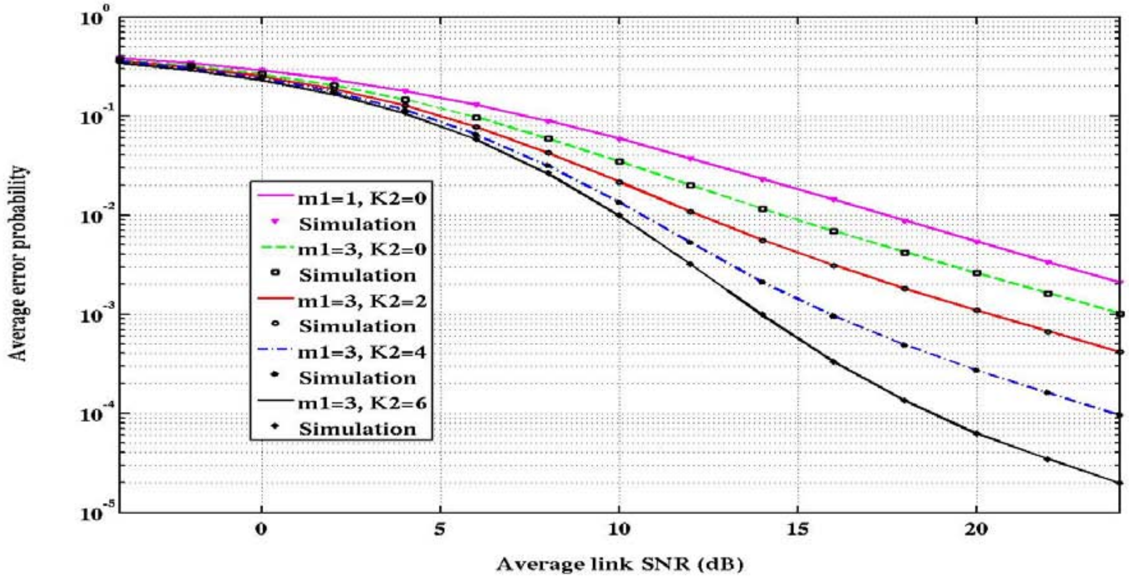


Numerical Results: Example set #3

Ergodic Capacity



Observations and Comments



1. While the differences in the ergodic capacity are modest, dramatic differences can be seen in the average error probability curves.
2. Performance is improved when the Rician parameter is increased.
3. The limiting slopes of the P_s curves are the same for the different cases of the K parameter.
4. There exists a substantial SNR gain resulting from increasing the fading parameter K.

Numerical Results: Example set #4

Dual-hop AF system

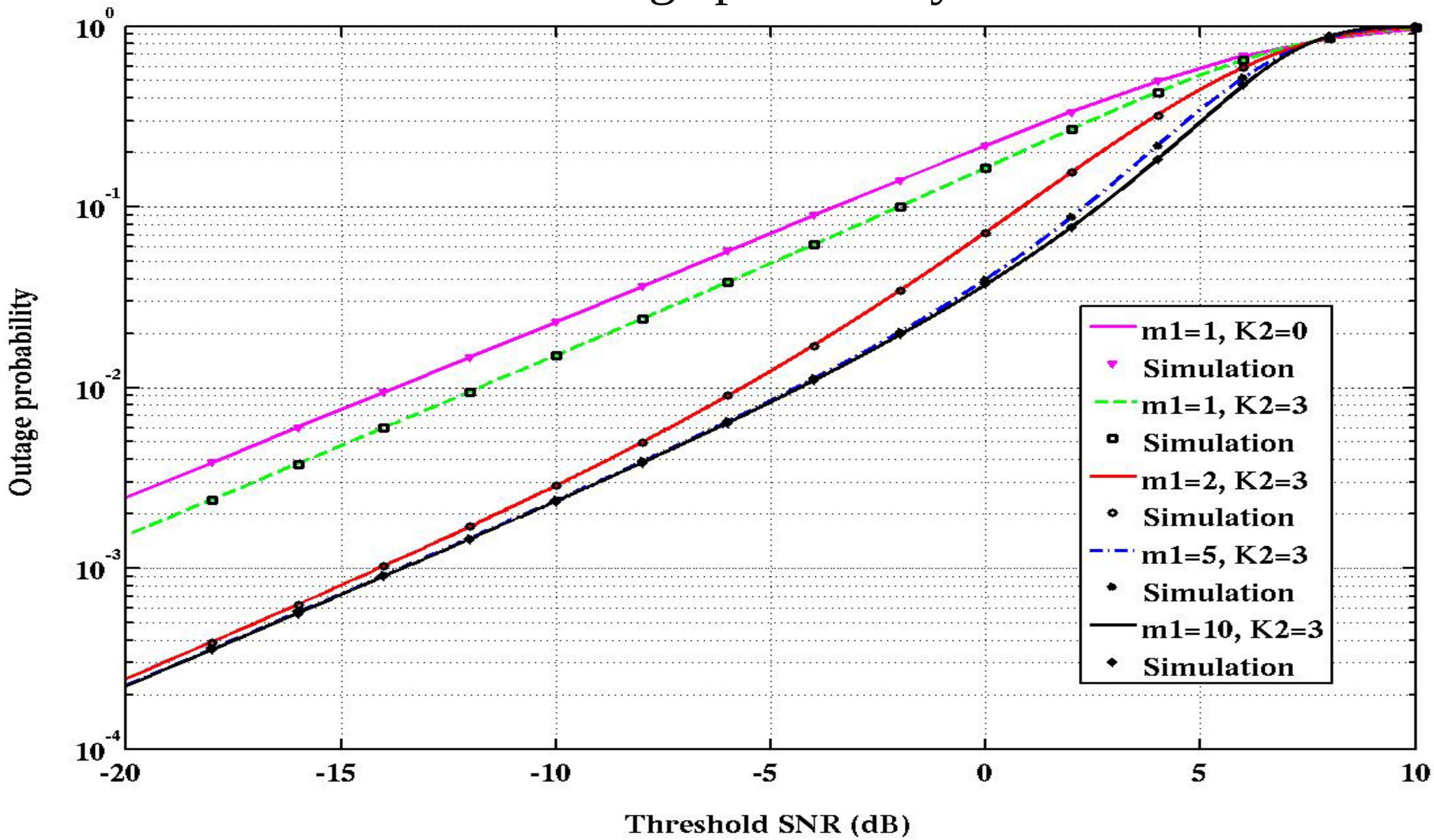
Mixed Nakagami-*m* and Rician Links

Variable *m* and fixed K

- ❑Case(1): $m_1=1, K_2=0$
- ❑Case(2): $m_1=1, K_2=3$
- ❑Case(2): $m_1=2, K_2=3$
- ❑Case(2): $m_1=5, K_2=3$
- ❑Case(2): $m_1=10, K_2=3$

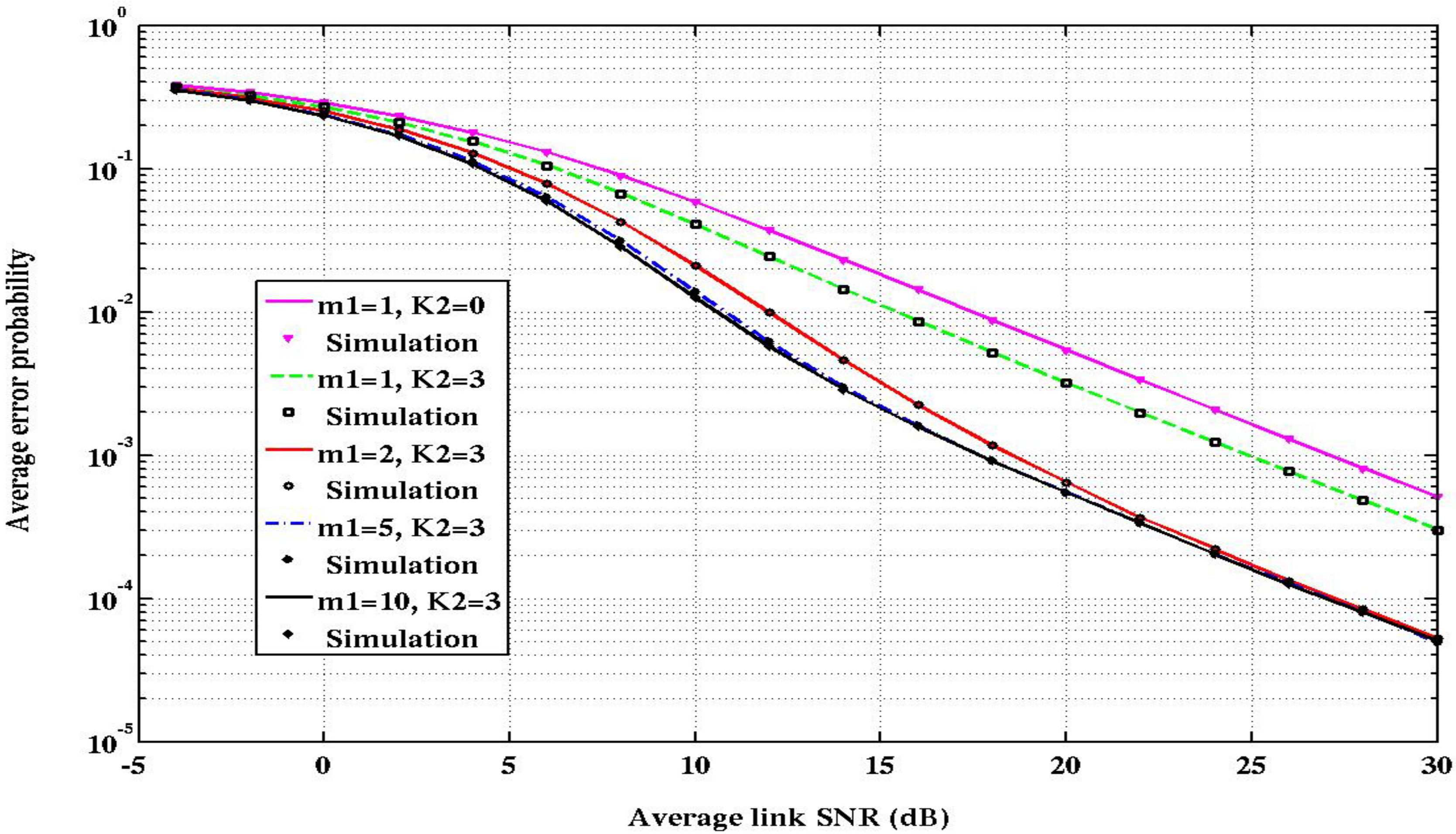
Numerical Results: Example set #4

Outage probability

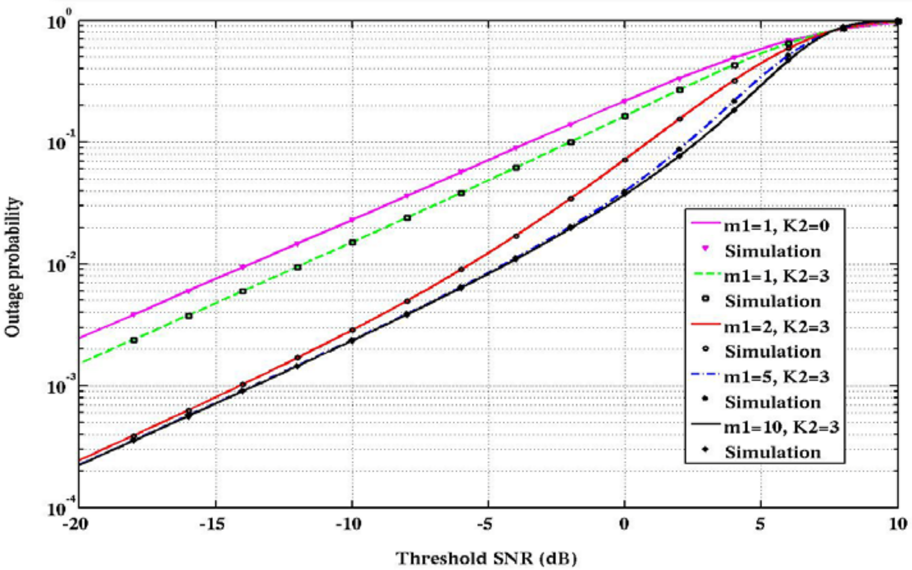


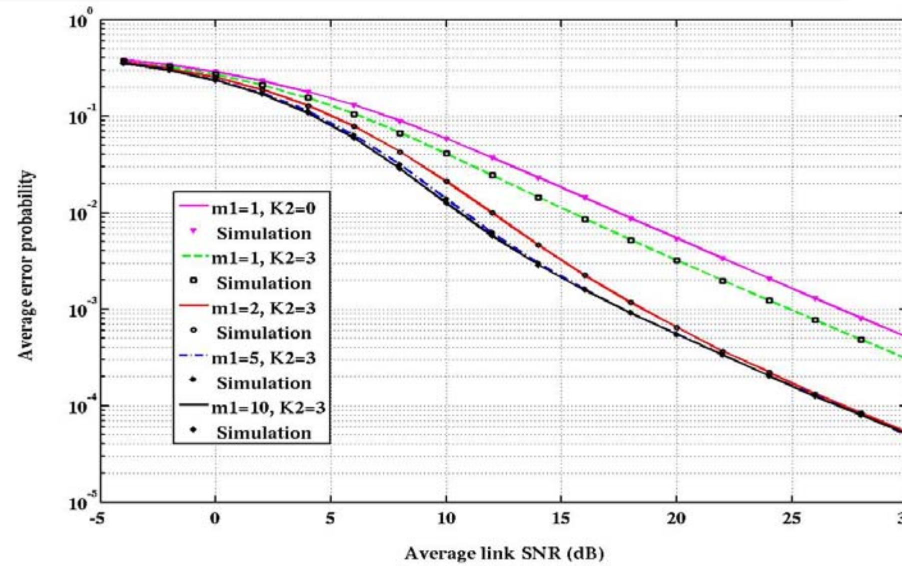
Numerical Results: Example set #4

Average error probability



Observations and Comments





1. Performance is improved when the Nakagami-*m* parameter is increased.
2. The limiting slopes of the P_s and P_{out} curves are the same for the different cases of the *m* parameter. → Not expected !!!

   → Presence of a Rician link is a bottleneck to the performance improvement.

3. There exists a negligible improvement in performance with increasing the fading parameter *m*.

   → Increasing *m* at fixed values of *K* has diminishing returns.

Outline

1. Introduction
2. System and Channel Models
3. Dual-Hop AF Systems
4. **Maximum End-to-End SNR Relay Selection**
5. Full Selection Dual-Hop AF Systems
6. Conclusion

Maximum end-to-end SNR relay selection

➢In this system, one of N available relay nodes is selected to relay the radio signal.

➢Selection is based on a maximum end-to-end SNR policy.

$$\max_{R_n \in \{R_1, \cdots, R_N\}} \gamma_{S-R_n-D} \Rightarrow \gamma_t$$

where,

$$\gamma_{S-R_n-D} = \frac{\gamma_{S-R_n} \gamma_{R_n-D}}{\gamma_{S-R_n} + \gamma_{R_n-D} + 1}$$

Maximum end-to-end SNR relay selection

Since the PDF and the CDF of γ_{S-R_n-D} are known, order statistics can be used to obtain the PDF and the CDF of γ_t.

And since γ_t is the maximum (N^{th} order statistic) of a set of N *i.i.d.* random variables, γ_{S-R_n-D}, then the CDF of γ_t is obtained as:

$$F_{\gamma_t}(r) = \left[F_{\gamma_{S-R_n-D}}(r)\right]^N$$

and the PDF is obtained as:

$$f_{\gamma_t}(r) = N f_{\gamma_{S-R_n-D}}(r) \left[F_{\gamma_{S-R_n-D}}(r)\right]^{N-1}$$

Numerical Results: Example set #5

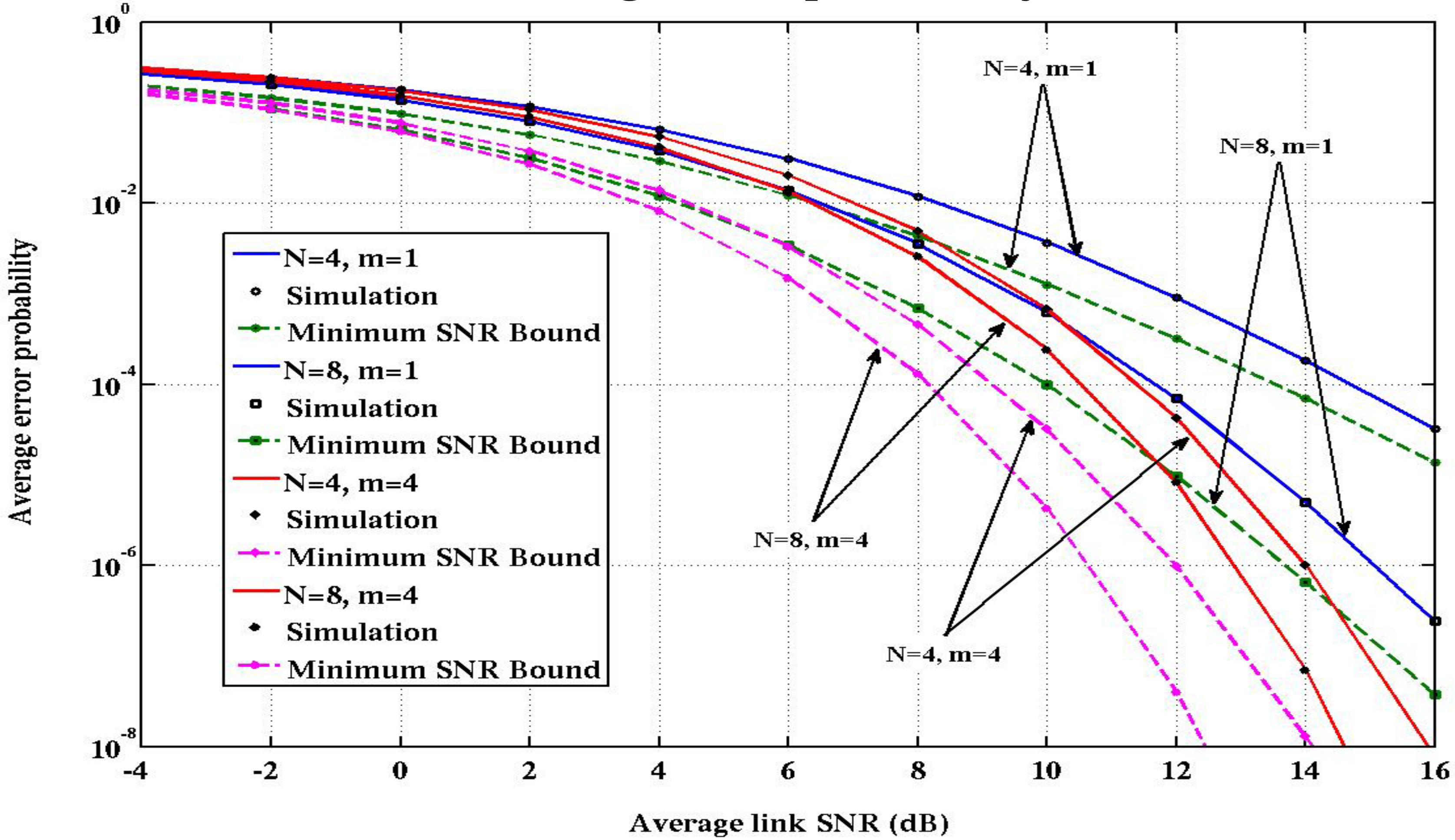
Maximum end-to-end SNR
Relay Selection AF system
Bound Vs Exact

- i.i.d. Rayleigh Links: *m=1 , N=4*
- i.i.d. Rayleigh Links: *m=1 , N=8*

- i.i.d. Nakagami-*m* links: *m=4, N=4*
- i.i.d. Nakagami-*m* links: *m=4, N=8*

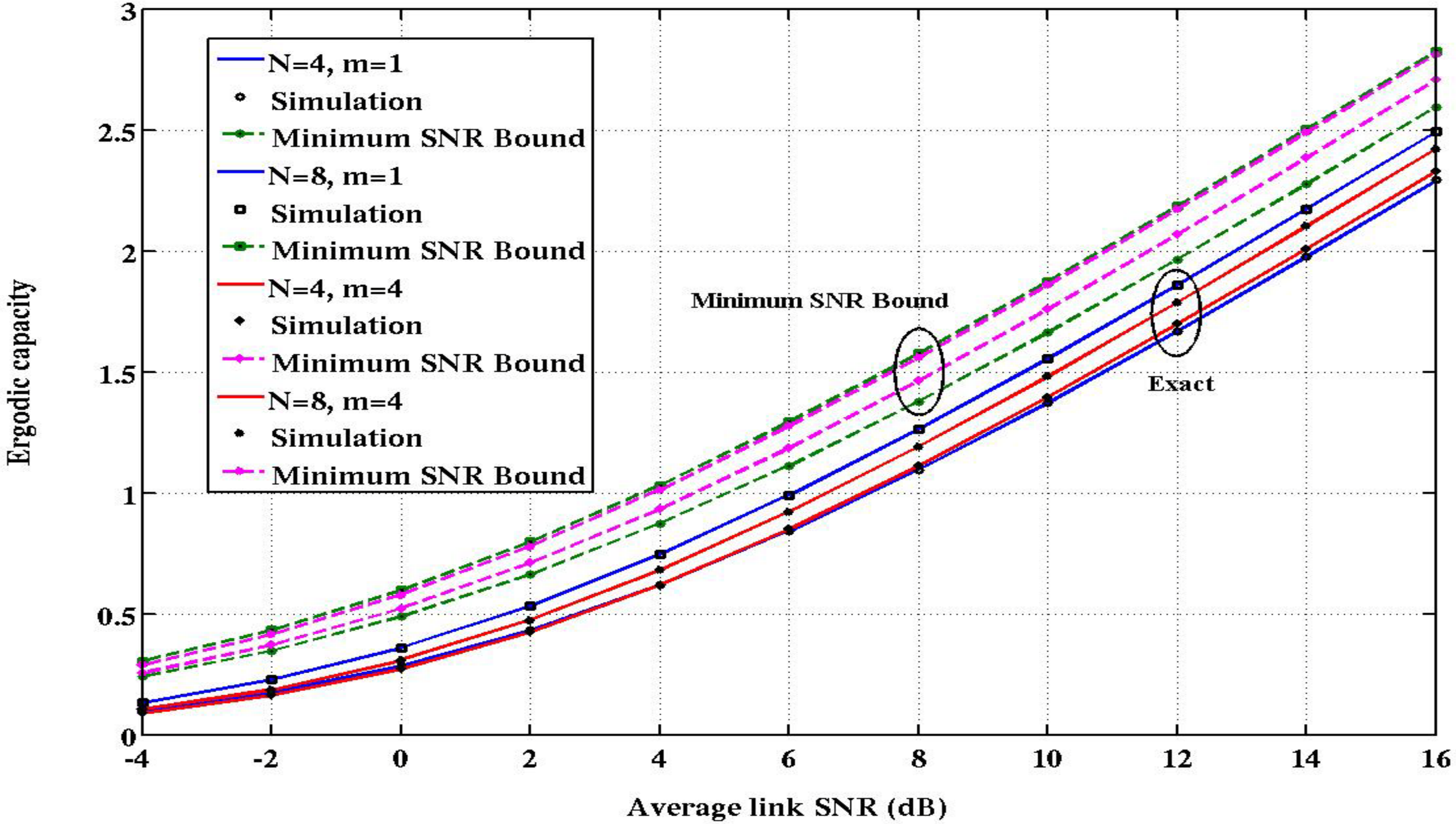
Numerical Results: Example set #5

Average error probability

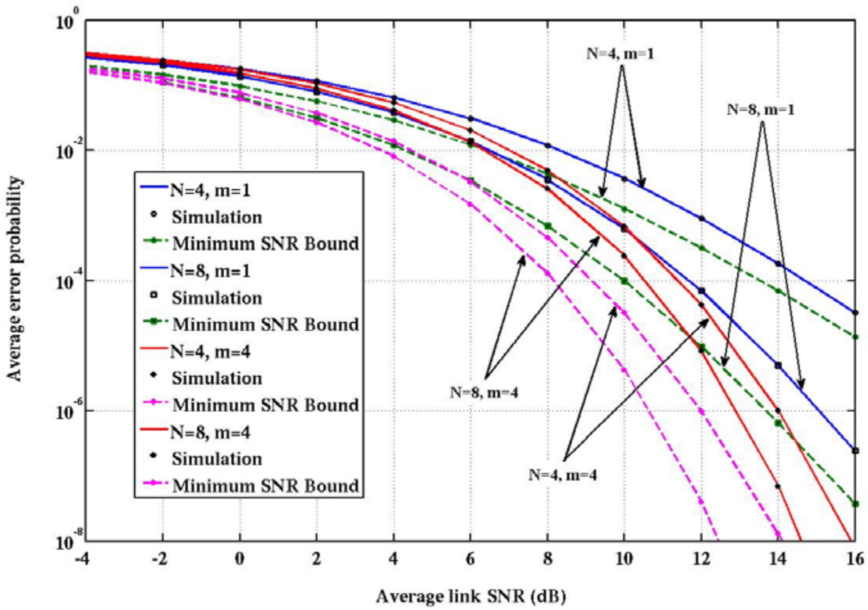


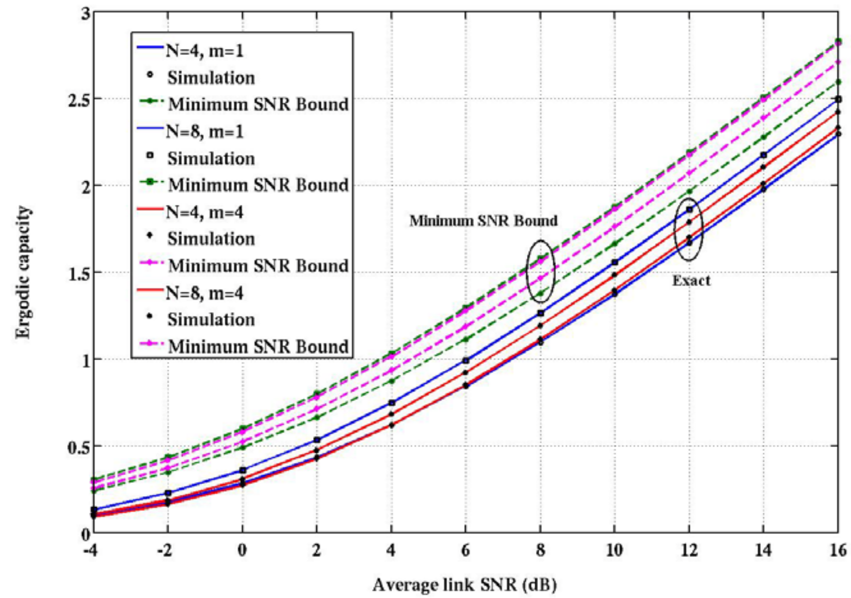
Numerical Results: Example set #5

Ergodic capacity



Observations and Comments





1. Loose bounds for all SNR ranges of interest.
2. Bound becomes more loose for higher values of *N*.

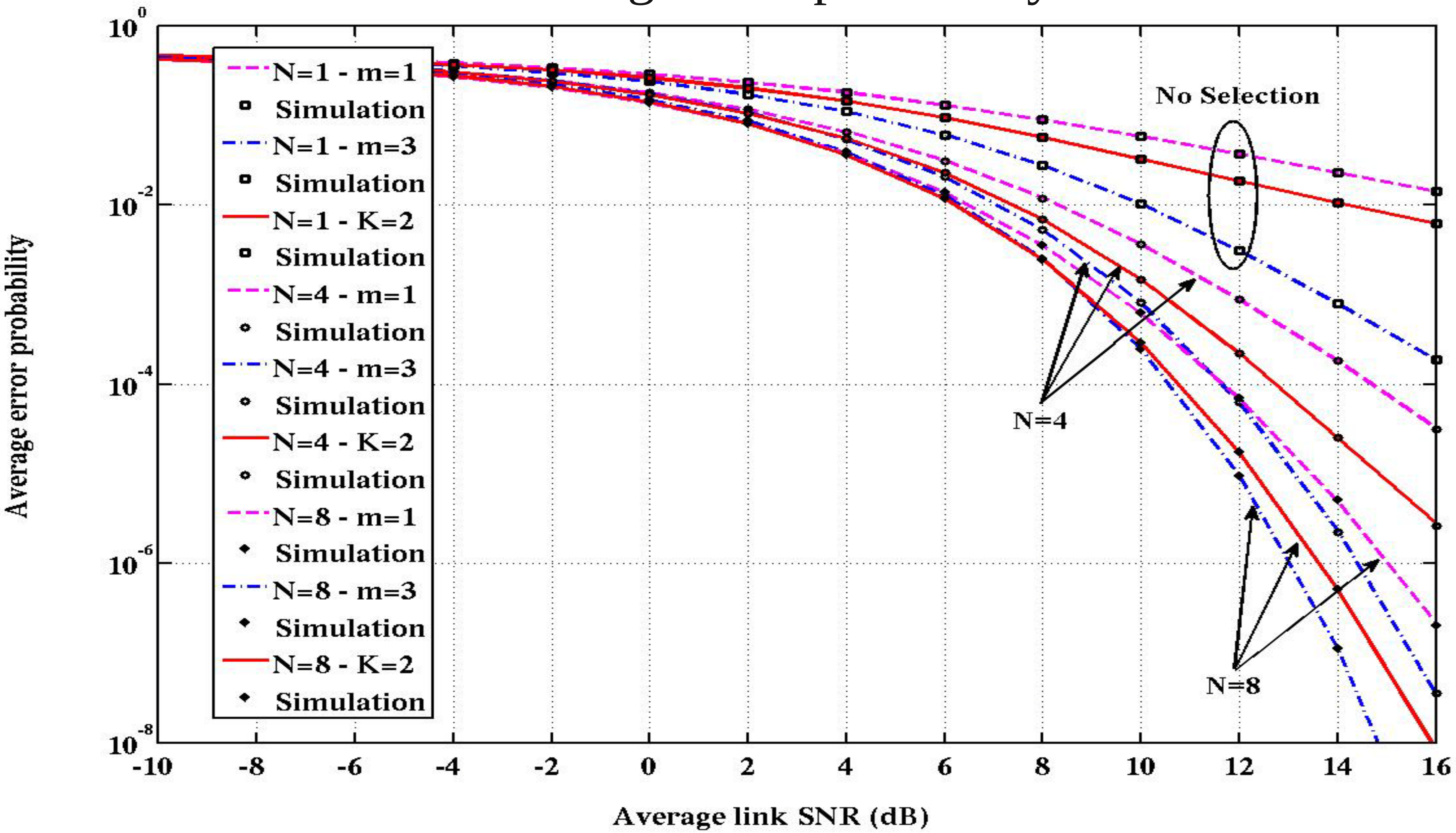
Numerical Results: Example set #6

Maximum end-to-end SNR
Relay Selection AF system
Effect of *N*

- i.i.d. Rayleigh Links: *m=1 , N=1, 4, 8*
- i.i.d. Nakagami-*m* Links: *m=3 , N=1, 4, 8*
- i.i.d. Rician Links: *K=2 , N=1, 4, 8*

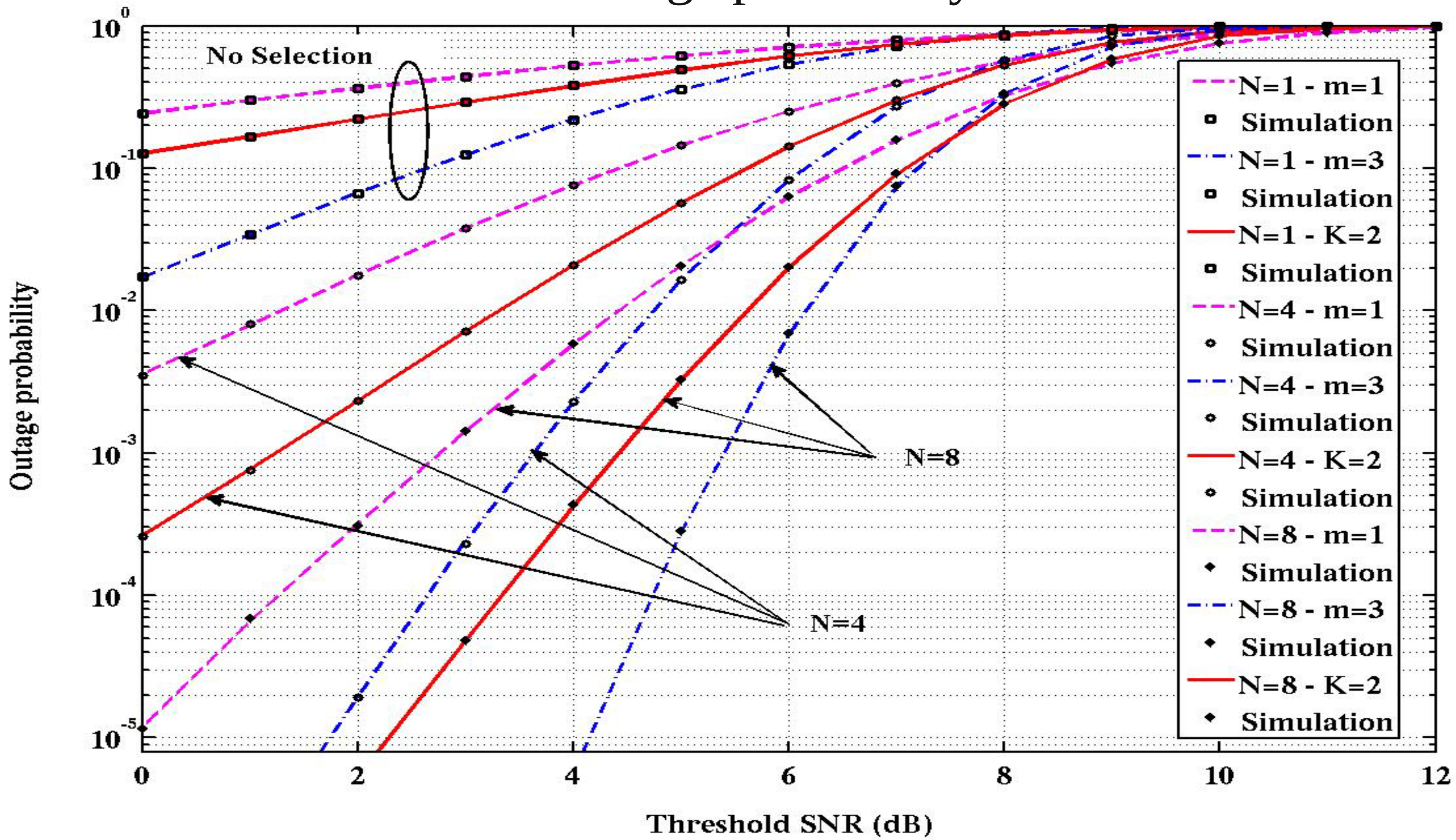
Numerical Results: Example set #6

Average error probability

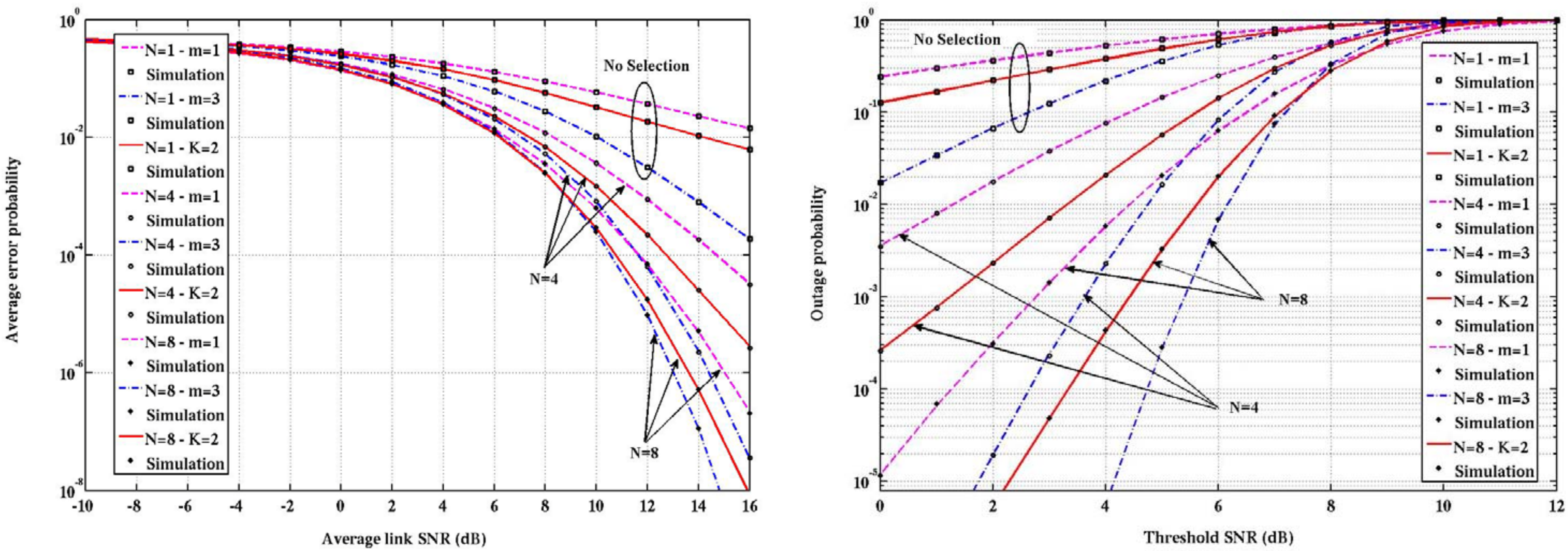


Numerical Results: Example set #6

Outage probability



Observations and Comments



1. Performance of dual-hop AF systems with relay selection is superior to that of dual-hop AF systems without relay selection.
2. Performance is improved with increasing N.
3. Limiting slopes of P_s and P_{out} curves are affected by and proportional to N.

Outline

Full selection dual-hop AF systems

➢In this system, either the direct single-hop path or one of the *N* dual-hop paths is selected as the communication path.

➢Selection is also based on a maximum end-to-end SNR policy.

$$\max\{\gamma_0, \gamma_m\} \Rightarrow \gamma_t$$

where,

$$\max_{n \in \{1,2,\cdots,N\}} \gamma_n \Rightarrow \gamma_m$$

where,

$$\gamma_n = \frac{\gamma_{S-R_n}\gamma_{R_n-D}}{\gamma_{S-R_n} + \gamma_{R_n-D} + 1}$$

Full selection dual-hop AF systems

Order statistics is used to determine the PDF and the CDF of the received end-to-end SNR as:

$$f_{\gamma_t}(r) = f_{\gamma_0}(r)F_{\gamma_m}(r) + F_{\gamma_0}(r)f_{\gamma_m}(r)$$

and,

$$F_{\gamma_t}(r) = F_{\gamma_0}(r)F_{\gamma_m}(r)$$

where the PDF, $f_{\gamma_m}(r)$, and the CDF, $F_{\gamma_m}(r)$, were previously obtained in closed-form.

Numerical Results: Example set #7

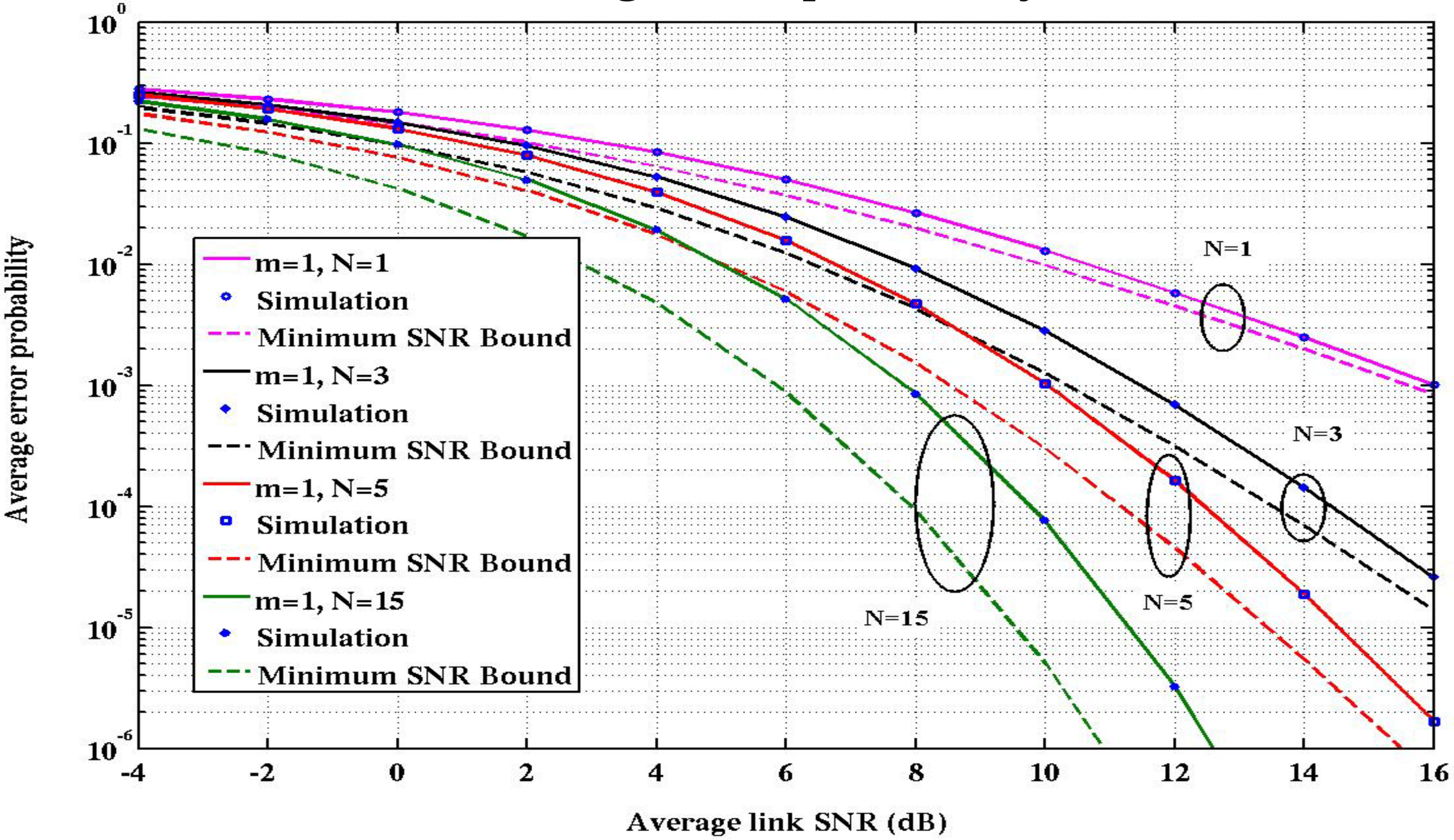
Full Selection AF system

Bound Vs Exact

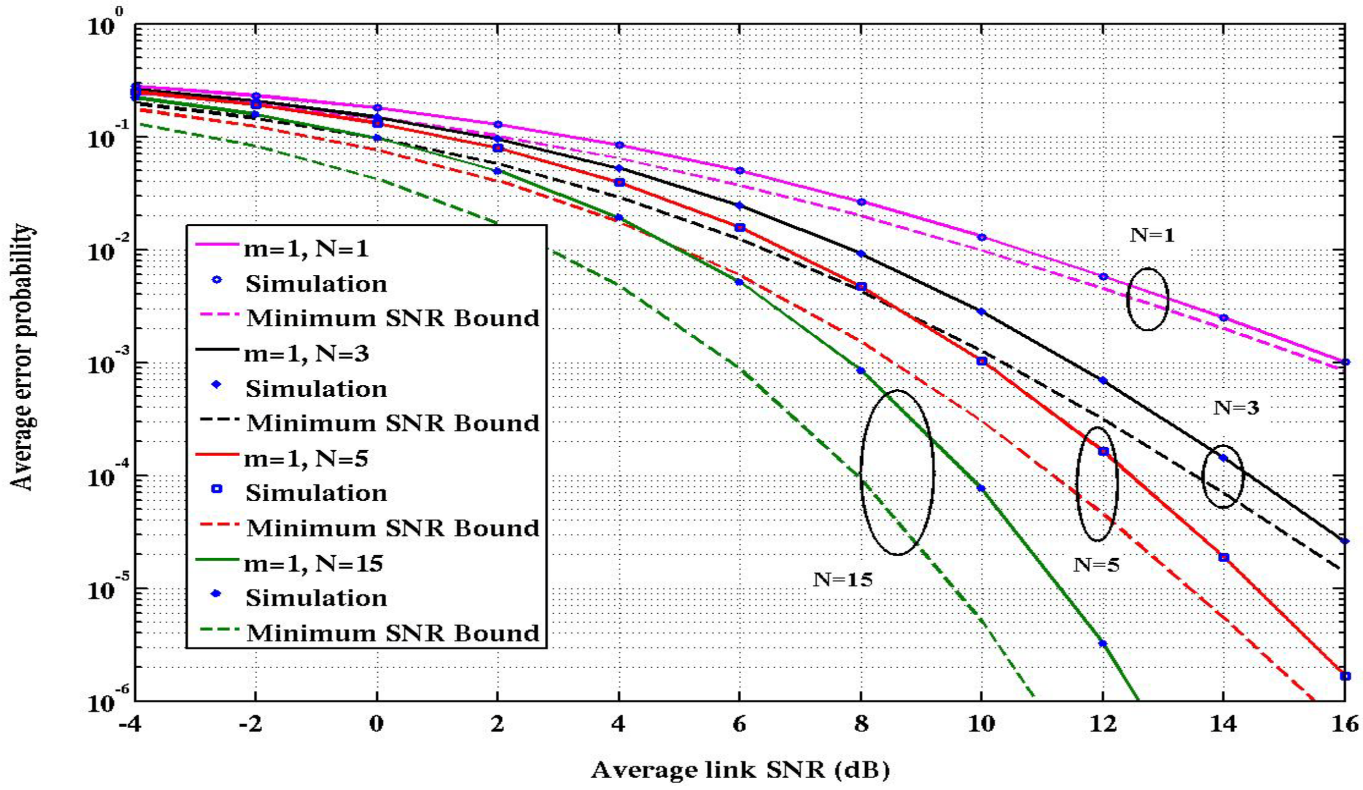
- i.i.d. Rayleigh Links: *m=1 , N=1, 3, 5, 15*

Numerical Results: Example set #7

Average error probability



Observations and Comments



1. Loose bounds for all SNR ranges of interest.
2. Bound becomes more loose for higher values of N.

Numerical Results: Example set #8

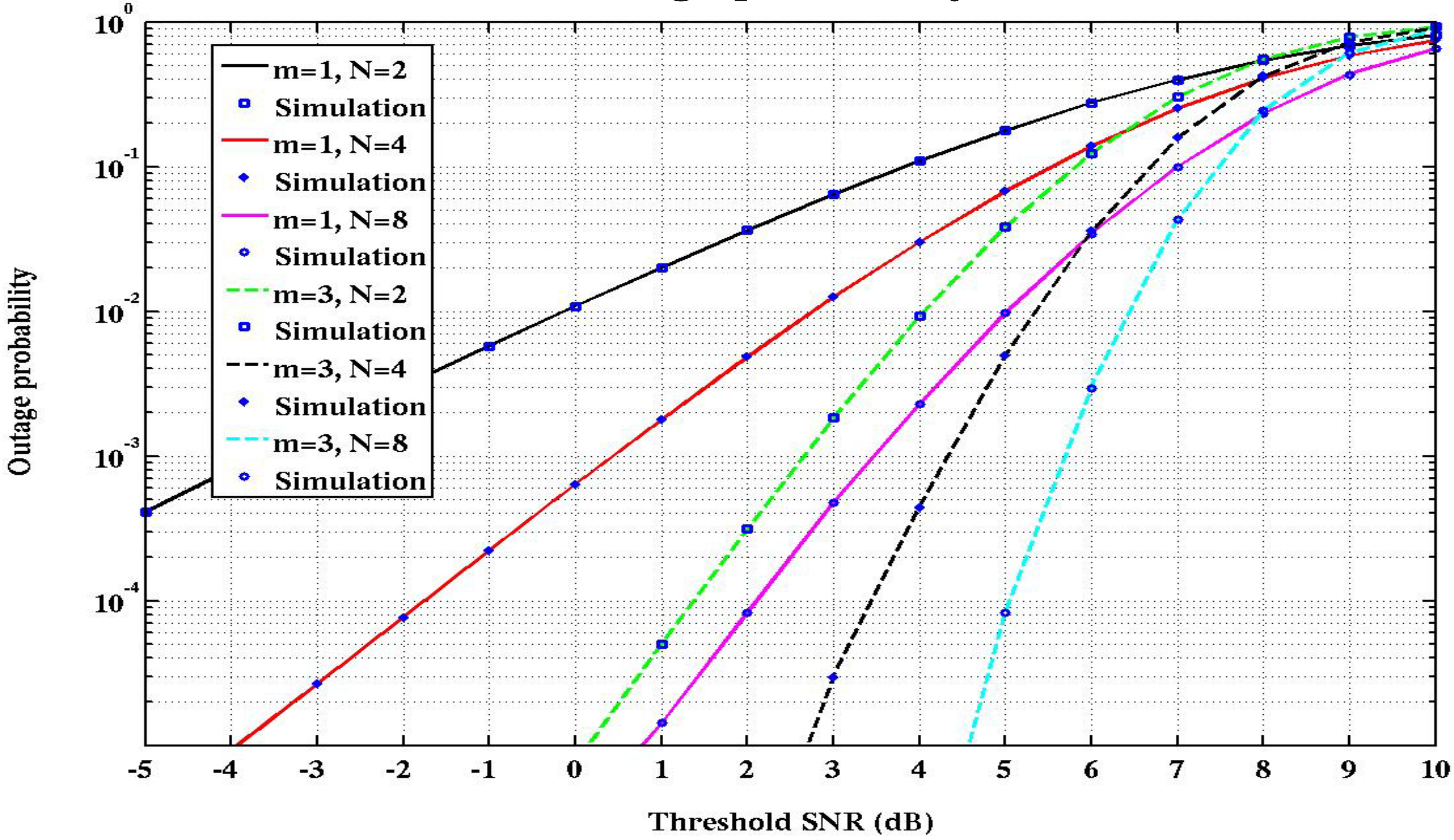
Full Selection AF system

Effect of N

- i.i.d. Rayleigh Links: $m=1$, $N=2, 4, 8$
- i.i.d. Nakagami-m Links: $m=3$, $N=2, 4, 8$

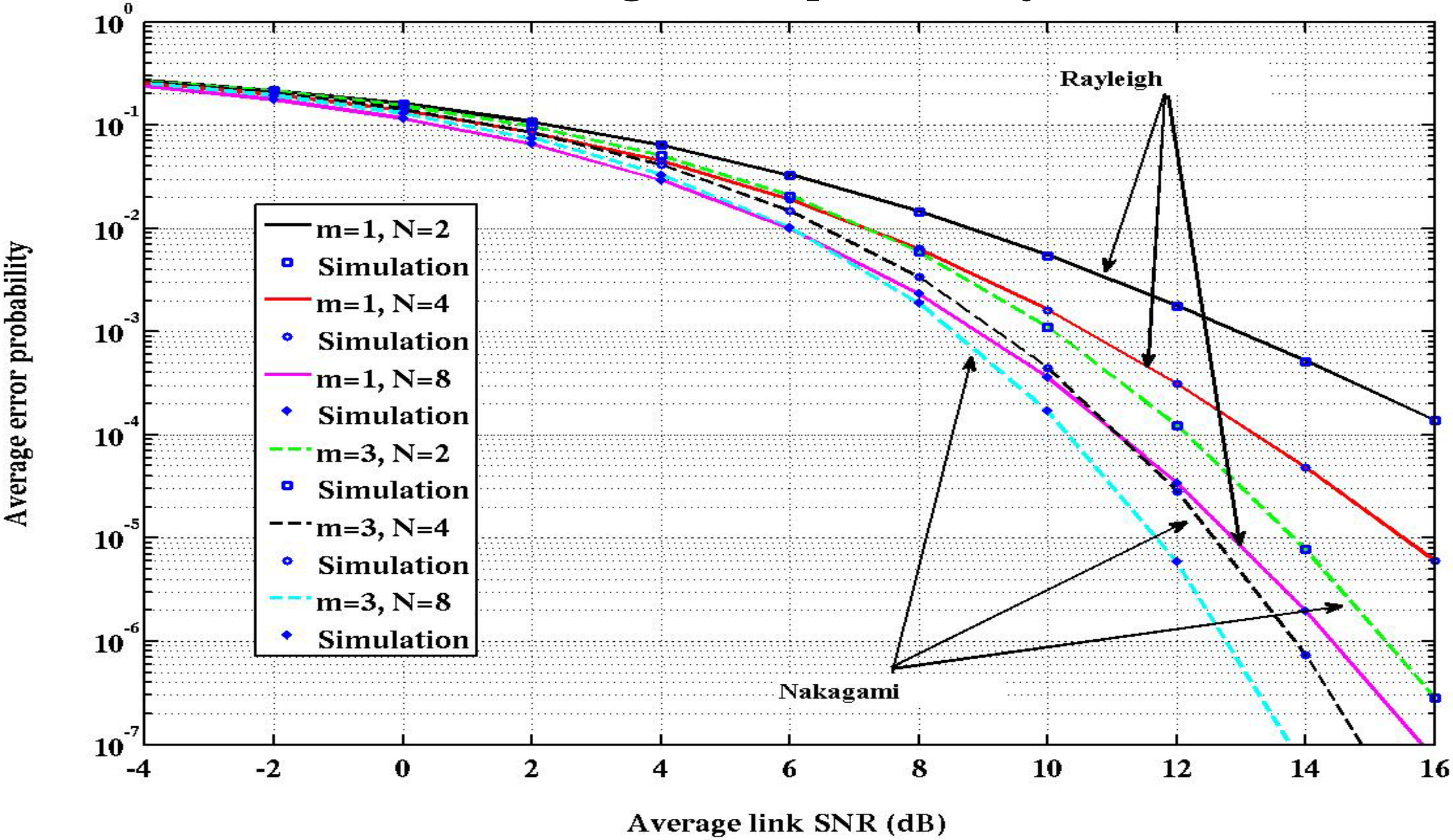
Numerical Results: Example set #8

Outage probability

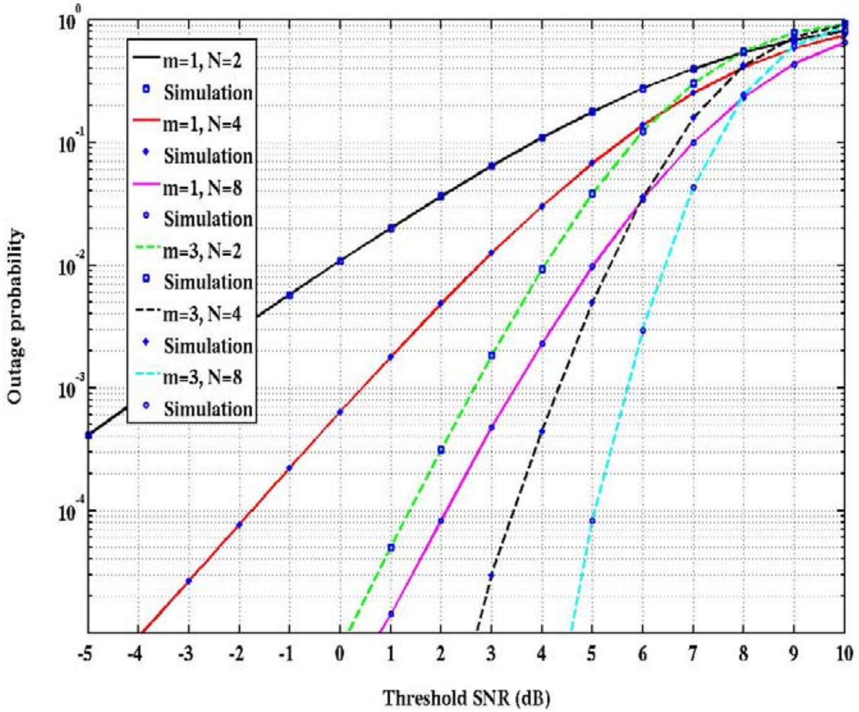


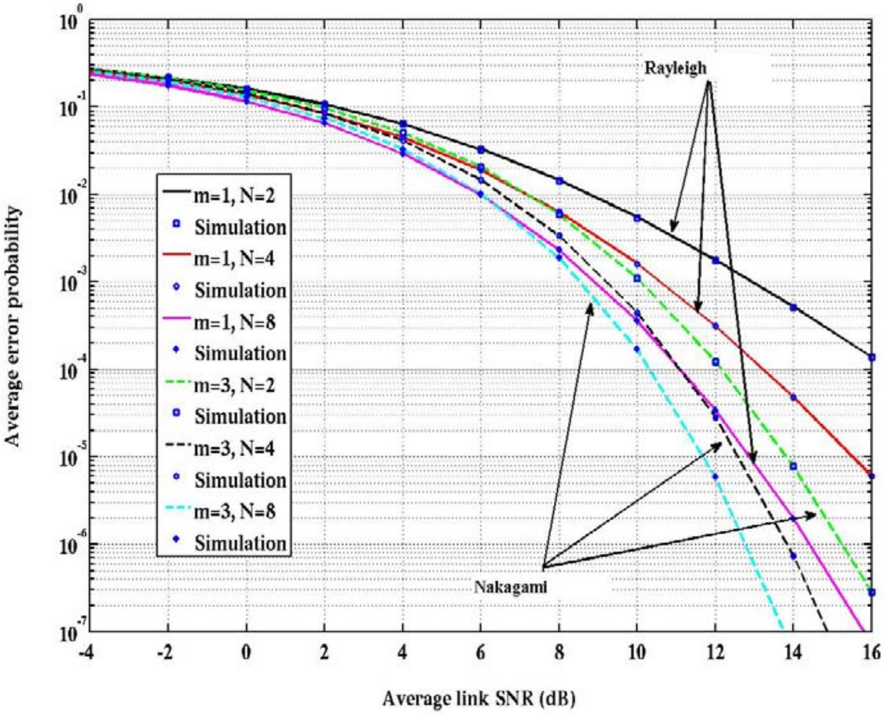
Numerical Results: Example set #8

Average error probability



Observations and Comments





1. Performance is improved with increasing *N*.
2. Limiting slopes of P_s and P_{out} curves are affected by and proportional to *N+1*.

Numerical Results: Example set #9

Direct path

Vs

Maximum end-to-end SNR selection

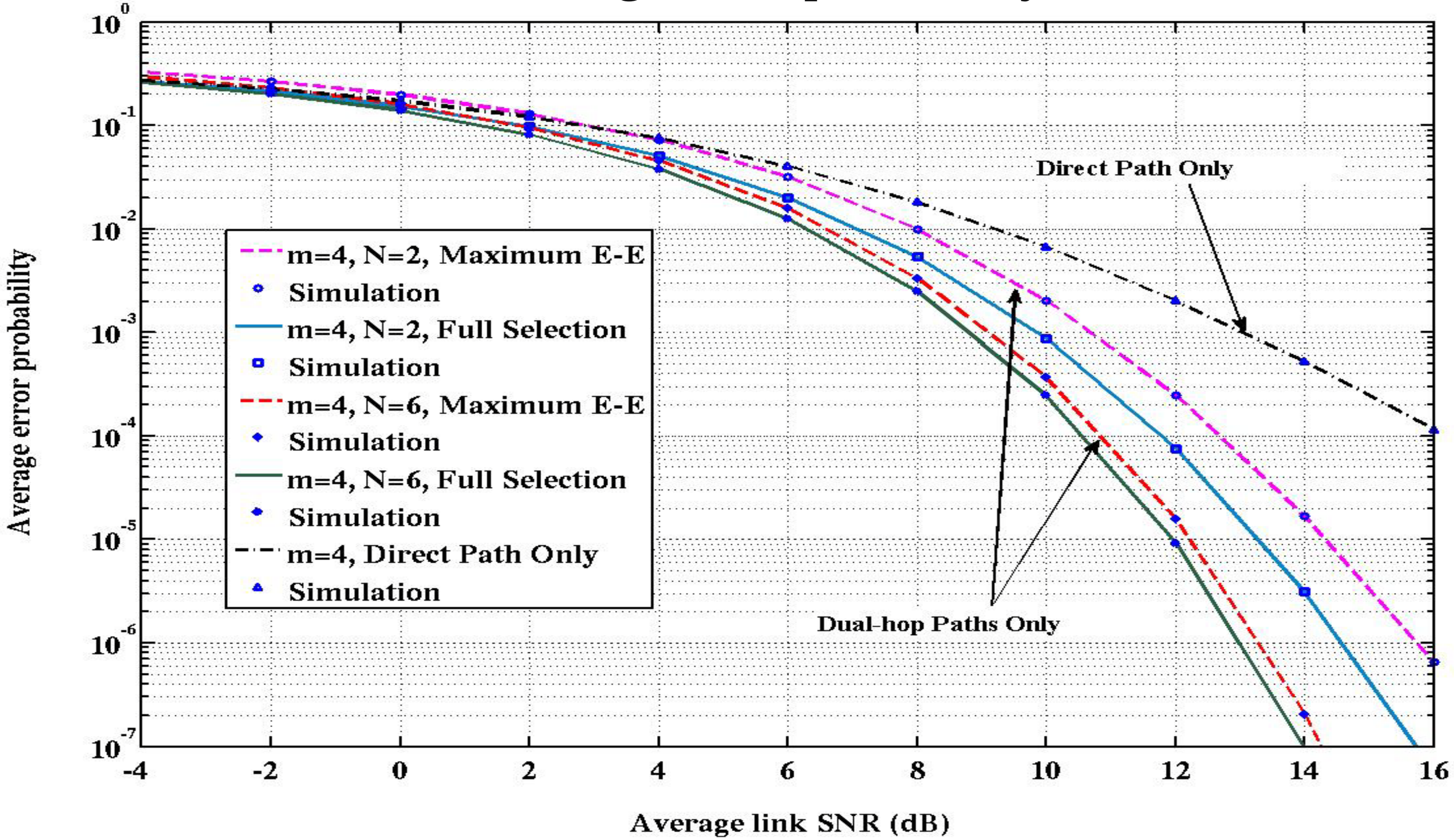
Vs

Full Selection AF system

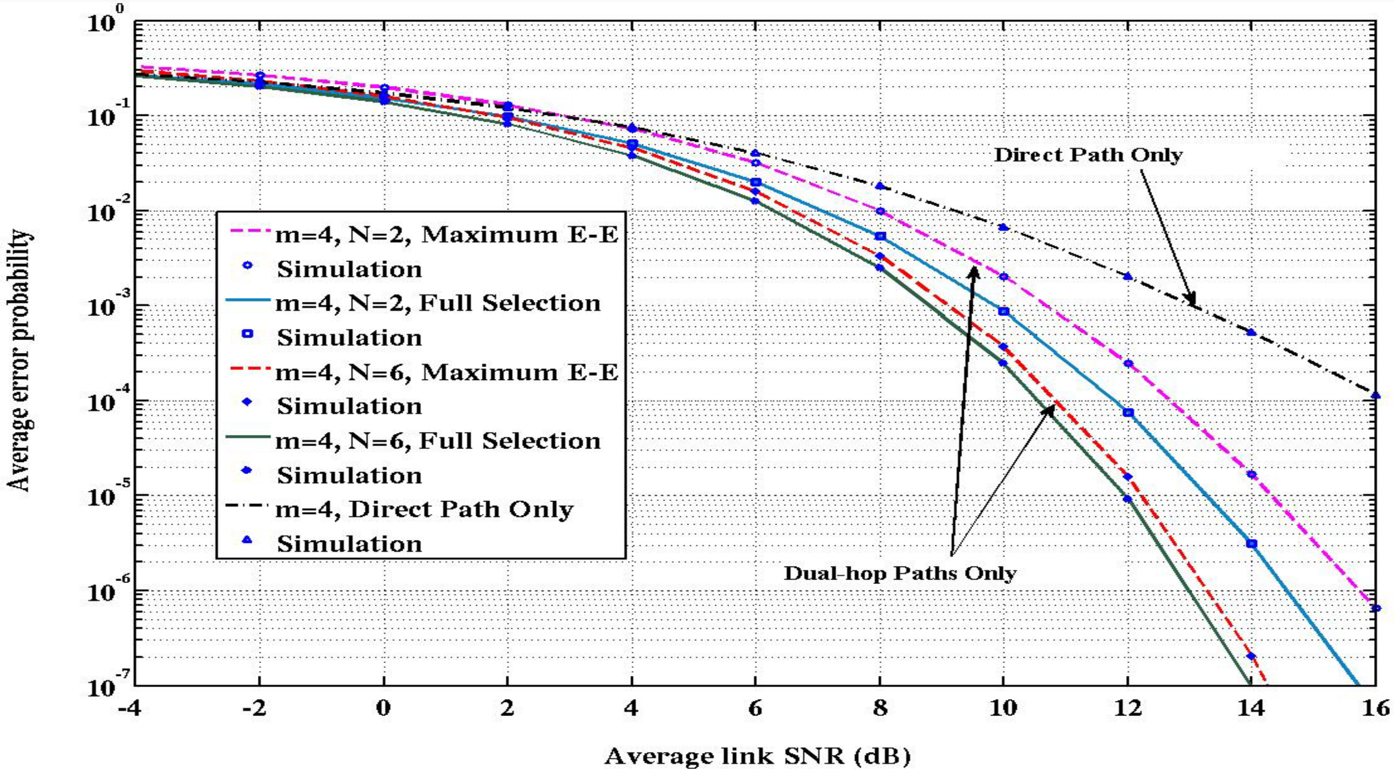
- i.i.d. Nakagami-m Links: $m=4$, $N=0,2,6$

Numerical Results: Example set #9

Average error probability



Observations and Comments



1. Performance of dual-hop AF systems with full selection is superior to that of dual-hop AF systems with maximum end-to-end SNR relay selection in the absence of the direct path link.

Outline

1. Introduction
2. System and Channel Models
3. Dual-Hop AF Systems
4. Maximum End-to-End SNR Relay Selection
5. Full Selection Dual-Hop AF Systems
6. Conclusion

Conclusion

- New theoretical analyses were presented to obtain exact, closed-form expressions for the PDF and the CDF of the end-to-end SNR in the cases of:

I. Dual-hop AF systems.
II. Dual-hop AF systems with maximum end-to-end SNR relay selection.
III. Dual-hop AF systems with full selection.

- Performance metrics such as ergodic capacity, outage probability and average error probability were obtained.

- Effects of the fading parameters and the numbers of relays were studied.

- Comparisons between the different systems were presented.

Questions

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Dual-Hop Mixed AF Systems: PDF

VALUES OF THE PDF FOR DIFFERENT LEVELS OF TRUNCATION (N)

$$(m_1 = 3, K_2 = 6)$$

N	$\overline{\gamma}_1 = \overline{\gamma}_2 = 1$ and $r = 1$	$\overline{\gamma}_1 = \overline{\gamma}_2 = 10$ and $r = 1$	$\overline{\gamma}_1 = \overline{\gamma}_2 = 10$ and $r = 10$
25	**0.01152831**1949557	**0.0480665598**80828	**0.005755227**844295
30	**0.011528312654**584	**0.0480665598967**75	**0.0057552279447**03
35	**0.011528312654876**	**0.048066559896781**	**0.005755227944740**

The table shows the number of decimal places that did not change, by adding more terms to the summations, in bold

→This indicates that a finite number of terms can be used for the summation with a negligible truncation error

Dual-Hop Mixed AF Systems: CDF

VALUES OF THE CDF FOR DIFFERENT LEVELS OF TRUNCATION (N)

$m_1 = 3, K_2 = 6$ AND $\overline{\gamma}_1 = \overline{\gamma}_2 = 10\, dB$

N	$\gamma_{th} = 0\, dB$	$\gamma_{th} = -6\, dB$	$\gamma_{th} = -10\, dB$
25	**0.01860131**5453518	**9.5477**20331142262 $\times 10^{-4}$	**2.54447**0348313510 $\times 10^{-4}$
30	**0.018601314**115370	**9.5477068**94568186 $\times 10^{-4}$	**2.54445691**0809068 $\times 10^{-4}$
35	**0.018601314114881**	**9.547706889654339** $\times 10^{-4}$	**2.544456905893000** $\times 10^{-4}$

The table shows the number of decimal places that did not change, by adding more terms to the summations, in bold

→This indicates that a finite number of terms can be used for the summation with a negligible truncation error